1 Instructor's Guide

1.1 Students' Task

In this improvisational skit, Kirk and Spock, (played by two students) are at different locations on a hostile planet and the Enterprise crew (the rest of the class) must devise a way to describe the distance between them in order to "beam" them to safety. Students use the distance formula $|\vec{r} - \vec{r'}|$ to find the distance between Captain Kirk and Mr. Spock. The role of position vectors and the origin in the calculation of coordinate and origin independent distance is revealed.

1.2 Introduction

We do this activity by having students act out a "Star Trek" skit. Other scenarios are, of course, possible. If you want to use the Star Trek scenario, reassure students who might not know about Star Trek series that all they need to know is that Kirk, Spock, and Scotty are officers on a starship. The starship has the ability to "beam" people around, i.e. move them from anywhere to anywhere via advanced technology called a transporter that requires lots of energy. This is the way the story goes:

Ask for volunteers to play the roles of Captain Kirk and Mr. Spock. The instructor takes the role of Scotty. (Stand on the table to indicate the you are orbiting in a spaceship.) Everyone else is a "red shirt" who wants to be promoted to a shirt of another color. The "red shirts" are the one-time characters who can be killed off without long-term consequences to the television series. To be promoted to a shirt of another color, students will have to impress Scotty with their calculational ability so that he will recommend them.

Kirk and Spock are in separate places in a city on the surface of a new planet. Kirk is under attack by aliens. But the ship is also under attack by aliens. The transporter is damaged and cannot be used to beam anyone far enough to beam them on board the spaceship, so Scotty must beam Spock to Kirk to rescue him. The main computer is down, so Scotty must set the transporter controls by hand. How can he figure out how far Spock is from Kirk so he can set the power levels on the transporter correctly?

Kirk and Spock have communicators so that they can talk to Scotty. Central question: How do they let Scotty know where they are? Fortunately, they can both look outside windows and see the same large red building (draw this on the board). Laser range finders and compasses on their communicators will let them know the distance and direction to the red building. With this information, the red shirts must calculate the distance.

Various red shirts volunteer to come to the board. Each volunteer draws/elaborates diagrams relevant to the scenario and/or calculates the next step in finding the distance between Kirk and Spock. Make sure to let many red shirts have an opportunity to participate. (Some institutions run the distance calculation as a small group activity so that more students can participate.)

1.3 Student Conversations

The main purpose here is to help students gain a geometric understanding of the geomertry of the distance formula: $|\vec{r} - \vec{r'}|$.

In order to say where something is, you must first say where it is with respect to a known something else–an origin, in this case the red building. This is the purpose of the position vector $\vec{r}$.

The position vector $\vec{r}_K$ represents Kirk's position and $\vec{r}_S$ represents Spock's position. We find it pedagogically useful to discuss and use mneumonic supscripts (K and S) on the vectors.

From the distances and directions, the red shirts should rewrite the position vectors $\vec{r}_K$ and $\vec{r}_S$ in an appropriate coordinate system using the formula $\vec{r} = x\hat{x} + y\hat{y}$. Add a $z\hat{z}$ if you want.

Emphasize that $\vec{r} - \vec{r'}$ is a vector. Review vector addition/subtraction: the components are given by:

$$\vec{r} - \vec{r'} = (x\hat{x} + y\hat{y}) - (x'\hat{x} + y'\hat{y}) \tag{1}$$
$$= (x - x')\hat{x} + (y - y')\hat{y} \tag{2}$$

Emphasize that $|\vec{r} - \vec{r'}|$ is a scalar, the length of the vector $\vec{r} - \vec{r'}$. You find the magnitude of any vector by taking the square root of the dot product of the vector with itself, i.e. $v = |\vec{v}| = \sqrt{\vec{v} \cdot \vec{v}}$.

Some students will want to avoid the complexities of using $|\vec{r} - \vec{r'}|$ by putting one point at the origin, thus effectively setting $\vec{r'}$ to zero. Point out that, while this is an excellent strategy when there are just two points in the problem, but fails if there are more.

MANY students will want to use the Pythagorean theorem. Emphasize that this is absolutely correct and then make them do it the vector way as well, so that they see that a PROOF of the Pythagorean theorem is the vector calculation they are doing.

1.4 Wrap-up

It's important to emphasize that ideally, one just stretches a tape measure between two objects to find the distance between them. When this is not possible, especially when you are trying to find a distance in an equation, it is necessary to go through all these steps: find the coordinates of each point with respect to some agreed upon origin (in this case, the large red building). A coordinate independent representation for the distance, common in many advanced physics texts, is $|\vec{r} - \vec{r'}|$, written in terms of the position vectors of the two points with respect to a common origin. Write the position vectors in an appropriate coordinate system, subtract, take the dot product and the square root. Students should understand the relationship between vector addition, the magnitude of a vector, the dot product, and the Pythagorean theorem.

It may interest some advanced students that while $|\vec{r} - \vec{r'}|$ is independent of coordinates and origin, $\vec{r}$ is independent of coordinates, but not independent of origin.

Student handout Find a coordinate independent expression for the distance between two points and then evaluate it in rectangular coordinates.