Student handout Use this *Mathematica* notebook or this GeoGebra applet to explore the time dependence of quantum states on a ring.

1 Instructor's Guide

1.1 Introduction

When students first open up *Mathematica* it is useful to walk them through the worksheet explaining the basic purpose of each step in the worksheet. It is not important to focus on the details at this point, but going through the steps in the worksheet helps keep some groups from getting stuck or just blasting through the worksheet without understanding what Mathematica is doing.

It may be helpful to review with students the meaning of the probability density since this is the primary quantity plotted in this worksheet.

1.2 Student Conversations

The most common student mistake is to apply the time dependence to the wavefunction as a whole rather than applying the time dependence to each energy eigenfunction. Because these animations are premade, this activity will not help to address this issue unless you point to where and how the time dependence is added in the code.

After students have some time to play with the worksheet and understand what is being calculated and what value is being plotted, it is useful to ask some of the following questions.

- What is being plotted?
- What causes the time dependence? How is that related to what you've seen in the case of spins?
- What do you expect to happen if you change the amplitudes in front of one of the components of the wave function?
- What do you expect to happen if you make one of the coefficients complex or imaginary?
- What is the difference between those wave functions that appear to be sloshing versus those that appear to be rotating?

1.3 Wrap-up

Discussing what patterns students observed is a good start to a wrapup. This conversation can often lead to discussions of which wavefunctions result in observable time dependence and which do not. It is useful to look at the case of two wave functions added together, since this shows the simplest time dependence.

Update this discussion This observation can be connected back to the time evolution of the two state system being sinusoidal with a frequency related to the energy difference between the two states.

$$|\Psi\rangle = a_m e^{-i\frac{E_m}{\hbar}t}|m\rangle + a_n e^{-i\frac{E_n}{\hbar}t}|n\rangle$$

$$\langle\phi|\Psi\rangle = a_m e^{-i\frac{E_m}{\hbar}t}\langle\phi|m\rangle + a_n e^{-i\frac{E_n}{\hbar}t}\langle\phi|n\rangle$$

$$\langle\phi|\Psi\rangle = e^{-i\frac{E_m}{\hbar}t}(a_m\Phi_m(\phi) + a_n e^{-i\frac{(E_n-E_m)}{\hbar}t}\Phi_n(\phi))$$

$$|\langle\phi|\Psi\rangle|^2 = e^{i\frac{E_m}{\hbar}t}(a_m^*\Phi_m^*(\phi) + a_n^* e^{i\frac{(E_n-E_m)}{\hbar}t}\Phi_n^*(\phi))e^{-i\frac{E_m}{\hbar}t}(a_m\Phi_m(\phi) + a_n e^{-i\frac{(E_n-E_m)}{\hbar}t}\Phi_n(\phi))$$

$$|\langle\phi|\Psi\rangle|^2 = e^{i\frac{E_m}{\hbar}t}e^{-i\frac{E_m}{\hbar}t}(a_m^* a_m\Phi_m^*(\phi)\Phi_m(\phi)) + a_m^* a_n e^{i\frac{(E_n-E_m)}{\hbar}t}\Phi_m^*(\phi)\Phi_n(\phi)) + a_n^* a_m e^{-i\frac{(E_n-E_m)}{\hbar}t}\Phi_n^*(\phi)\Phi_m(\phi) + a_n^* a_n e^{i\frac{(E}{}}$$

If a_m and a_n are real then

$$|\langle\phi|\Psi\rangle|^2 = a_m^2|\Phi_m(\phi)|^2 + 2a_m a_n\cos\big(\frac{\Delta E_{mn}}{\hbar}t\big)\Phi_m(\phi)\Phi_n(\phi) + a_n^2|\Phi_n(\phi)|^2$$

It it is sometimes helpful to have a follow up discussion after students have had some time to play with the worksheet outside of class. This gives each student some time to be in control of the worksheet and gives them time to play without the distractions and pressures of class.

1.4 Extensions and Related Materials

This activity can be used as part of a sequence of Mathematica activities that allows one to explore probability densities for a particle first confined to a ring, then to the surface of a sphere, and finally for the entire three-dimensional hydrogen atom.

Related homework question: Particle on a Ring Time Dependence: Mathematica Notebook