Student handout Consider a quantum particle on a ring. At $t = 0$, the particle is in state:

$$|\Phi(t=0)\rangle = \frac{7i}{10}|-2\rangle - \frac{1}{2}|-1\rangle + \frac{1}{2}|0\rangle - \frac{1}{10}|2\rangle$$

1. Find $|\Phi(t)\rangle$
2. Go to the Ring States GeoGebra applet on the course schedule. Explore changing values of initial coefficents in the applet and see how $Re(\psi)$, $Im(\psi)$, and $|\psi|^2$ change with time.
   Then, create the state given above. How do both pieces of the wavefunction and the probability density change with time?
3. Calculate the probability that you measure the z-component of the angular momentum to be $-2\hbar$ at time t. Is it time dependent?
4. Calculate the probability that you measure the energy to be $\frac{2\hbar^2}{I}$ at time t. Is it time dependent?

1 Instructor's Guide

1.1 Introduction

This activity is about calculating probabilities with states that depend on time. It is a good idea to remind students how to time evolve a state by giving students the initial state (that is on the handout) and ask them to write $|\psi(t)\rangle$ on small whiteboards. No time dependent probabilities will appear here, but they will see time dependent wave functions. In combination with Part 2 the goal is to drive home that probabilities will be time independent when they correspond to measurements of operators which commute with the Hamiltonian and be time dependent when they correspond to other operators which don't commute with the Hamiltonian.

1.2 Student Conversations

- **Degeneracy:** Students may experience some difficulty due to the degeneracy of some states, in particular that you are calculating probability of measuring a certain value, you have to include all the states that share that eigenvalue.
- **Notation:** Some will state without showing that the energy and angular momentum probabilities do not change with time. Ask them to calculate this explicitly to make sure that everyone in the group understands why (because the time-dependent phases norm square to 1 and there are no cross terms remaining). Understanding this calculation makes the comparison to position probability much easier - you see the cross terms go away for energy and angular momentum.
- **Stationary State:** Energy eigenstates have probability densities that don't change with time. The wavefunction changes with time, but the probability density doesn't.

- **possible additional prompt**: Does the probabilty density change with time if you have a state that is a superposition of $|0\rangle$ and $|1\rangle$?
- **possible additional prompt**: Can you find any states made up of two non-zero sliders that is stationary?

1.3 Wrap-up

- Remind students how to deal with degeneracy.
- Students will come out of seeing time independent probabilities for energy and angular momentum and want to reconcile that with seeing the moving wavefunctions and probability densities which DO depend on time in the 2nd part of this activity. Use this as a jumping off point or teser for part 2.

1.4 Extensions and Related Material

This is part 1 of a 2 part activity. Doing Part 2 in close proximinity or ideally in the same day is reccamended so students can see the parallels between probabilities which depend on time and those that do not.

Students readily grasp the strategy of finding probability amplitudes "by inspection" when they are given an initial state written as a sum of eigenstates. We find that students then find it extremely difficult to find probability amplitudes of wavefunctions that are not written this way (i.e. using an integral to find the expansion coefficients of a function). This activity should be followed up with another activity and/or homework from the Quantum Ring Sequence that allows students to practice this more general method.