Student handout Consider the following normalized abstract quantum state on a ring:

$$\Phi(\phi) = \sqrt{\frac{8}{5\pi r_0}} \cos^3(2\phi) \tag{1}$$

1. If you measured the z-component of angular momentum, what is the probability that you would measure $2\hbar$? $-3\hbar$?
2. If you measured the z-component of angular momentum, what other possible values could you have obtained with non-zero probability?
3. If you measured the energy, what possible values could you have obtained with non-zero probability?
4. What is the probability that the particle can be found in the region $0 < \phi < \frac{\pi}{2}$?

1 Superposition States for a Particle on a Ring: Instructor's Guide

This activity is the same as homework Superposition States for a Particle on a Ring

1.1 Introduction

If the previous activities (Energy and Angular Momentum for a Quantum Particle on a Ring and Time Dependence for a Quantum Particle on a Ring Part 1) have been done, little introduction is needed. It might be helpful to ask a small whiteboard question to help them remember what the eigenfunctions for a particle on a ring are.

In many cases, students will not think to rewrite the function as a linear combination of eigenstates. Even if they do, many students will have forgotten how. Therefore, you might start this activity in class and have students finish the calculations for homework.

1.2 Student Conversations

- **Finding Coefficients:**
  - Students readily grasp the strategy of finding probability amplitudes "by inspection" when they are given an initial state written as a sum of eigenstates. We find that students then find it extremely difficult to find probability amplitudes of wavefunctions that are not written this way (i.e. using an integral to find the expansion coefficients of a function).
  - If the students write the cosine in terms of exponentials, they can find the coefficients without using an integral. Group that recognize this will often finish early, so having them go back and do the integrals. Remind them that, while writing things in terms of exponentials is a

great strategy, knowing how to perform the integrals is the method that will work in any instance.

- Remind the students that the sum of the square of the norm of the coefficients they find should add to one for a normalized quantum state.

- **Degeneracy:** Students may experience some difficulty calculating probabilities due to the degeneracy of some states, in particular, that you have to include all the states that share that particular eigenvalue.

$$P_{E=\frac{m^2\hbar^2}{2I}} = |\langle m|\psi\rangle|^2 + |\langle -m|\psi\rangle|^2$$

1.3 Wrap-up

Use their work to demonstrate how finding all of the probabilities allows you to rewrite the wavefunction as a linear combination of eigenstates.

$$P_{L_z=m\hbar} = |\langle m|\Psi\rangle|^2 = \left|\int_{-\infty}^{\infty} \Phi_m^*(\phi)\Psi(\phi)\, d\phi\right|^2 = |c_m|^2$$

$$|\Psi\rangle = \sum_m c_m|m\rangle \doteq \sum_m c_m \left(\frac{1}{\sqrt{2\pi r_0}} e^{im\phi}\right)$$

1.4 Extensions and Related Materials

This is a part of Quantum Ring Sequence of activities.
Associated Homework Problem: QM Ring Function