1 Instructor's Guide

1.1 Introduction

Start with a short review of complex numbers in rectangular $x+iy$ and exponential $re^{i\phi}$ forms. Include an introduction to the graph of the complex plane (Argand diagram). Be aware that, while some students may have seen this content as early as high school, other students may only know that $i^2 = -1$. Discuss basic complex number algebra (addition and subtraction, multiplication, and division). See for example the content in the first six sections of LinAlg.

Draw the complex plane on a board at the front of the room and have students stand facing the right-hand side of the room. Introduce students to the idea of using their left arm as an Argand diagram where their left shoulder is the origin. Ask them to sweep their left arm around in a circle and show them that their arm aligns with the complex plane on the board. Point out that when their arm is parallel to the ground and in front of the student, the number represented is pure real and positive. When their arm is perpendicular to the ground and above the head of the student, the number represented is pure imaginary. Have students practice by showing some simple complex numbers, e.g. 1 and $-i$.

Now show the students a written prompt, have them close their eyes and act out the given complex number. Complex numbers can be given in both rectangular $x + iy$ and exponential $re^{i\phi}$ forms. After each prompt, have students open their eyes and compare. Discuss as necessary.

1.2 Student Conversations

This activity is particularly useful in helping the instructor identify which students have only minimal background on complex numbers and may need extra practice.

- Individual representations of complex numbers
  - 1, i, $-i$: These three examples help to orient the students to the representation and very few have issues.
  - $-3i$: This example brings up the problem of representing length when one's arm is only so long.
  - $e^{i\pi/4}$: This example is when some students may begin to have trouble. Remind them of the exponential ($re^{i\phi}$) form of complex numbers where r represents the length and ϕ the angle in the complex plane.
  - Multiply the previous complex number by $e^{i\pi/2}$: Introduce the word "phase" for a complex number of unit norm, i.e. $r = 1$. This example is intended to allow students to recognize that multiplication by a phase results in a rotation in the complex plane.
  - Multiply by i: This example is mathematically the same as the previous one, but conceptually harder. Emphasize that to multiply two complex numbers it may be easiest to first write them in exponential form. Discuss that multiplication by i, in particular, is a rotation by $\frac{\pi}{2}$ in the complex plane.
  - Find the complex conjugate of the previous complex number: Emphasize that complex conjugation is reflection in the real axis BECAUSE the phase goes to the negative of itself.

1.3 Wrap-up

Short class discussions are encouraged following each example before moving onto another complex number or operation. These examples can lead into a mini-lecture about when to use a particular form of a complex number, i.e. rectangular form for addition and, often, exponential form for multiplication and division.

Student handout Use your *left* arm to represent the complex plane, with your left shoulder representing the origin. For each complex number or operation given to you by your instructor, move your arm so that it points to the appropriate complex number.
The prompts are deliberately not given here, as they are best given one-by-one during class. A summary of the prompts appears in the solution.