1 Instructor's Guide

This activity is part of the Arms Sequence for Complex Numbers and Quantum States. If you have not used previous activities in the sequence, you may want to start with the introduction and a few of the prompts as listed in the first activity: Using Arms to Visualize Complex Numbers (MathBits).

1.1 Before the Activity:

Give a brief introductory lecture:

- In spin 1/2 systems, states are represented by a pair of complex numbers, subject to two restrictions:
  - The overall norm of the state must be 1, and
  - the overall phase does not affect the state.
- The set of states is known as the Bloch Sphere.

1.2 Prompt

Have the students pair up and write a state on the board.
Have them close their eyes and act out the given pair of complex numbers.
Complex numbers can be given in both rectangular $x + iy$ and exponential $re^{i\phi}$ forms.
After each prompt, have students open their eyes and compare. Discuss as necessary.

1.2.1 States

- First ask the students to show a particular spin 1/2 state, specified by a normalized pair of complex numbers. For example:

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ e^{\frac{i\pi}{3}} \end{pmatrix}$$

- Next, ask the students to show ALL the states equivalent to this one. Each pair of students should rotate their arms simultaneously, by the same amount.
- Then, ask students to show any different state. Bring out the fact that the relative angle between the two students arms must now be different.
- If your students have already learned how to represent the states that are spin up or down in the z, x, and y directions, all represented in the z-basis, i.e.

$$
\begin{array}{lcc}
S_z: & \begin{pmatrix}1\\0\end{pmatrix} & \begin{pmatrix}0\\1\end{pmatrix} \\
S_x: & \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} & \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} \\
S_y: & \frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix} & \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix}
\end{array}
$$

then, have them represent each of these states with their arms.

1.3 Wrap-up

Emphasize that it is the relative phase of a quantum state distinguishes different states. Two vectors that are only different by an overall phase describe the same quantum state. Of course, the relative magnitudes of the two components also matters.

1.4 Related Homework

- Phase
- Phase 2