1 Instructor's Guide

This activity is part of the Arms Sequence for Complex Numbers and Quantum States. If you have not used previous activities in the sequence, you may want to start with the introduction and a few of the prompts as listed in the first activity: Using Arms to Visualize Complex Numbers (MathBits) and a few of the prompts as listed in the third activity Using Arms to Represent Overall and Relative Phase in Spin 1/2 Systems.

This is a *discovery* activity that prepares students to understand the solution of the Schrodinger Equation.

1.1 Prompts:

1. **Set-Up**
   - Students should form pairs to represent a quantum statem.
   - Each student represents one of the S_z basis states:

     person on the left $\rightarrow |+\rangle$
     person on the right $\rightarrow |-\rangle$

   - Each student uses their left arm to represent the complex probability amplitude for their basis ket.

2. **Optional Warm-Up** Have students represent the following states: $|+\rangle_x$, $|-\rangle_x$, $|+\rangle_y$, $|-\rangle_y$. This will help students recognize these states when they come up later in the activity.

3. "Represent the state $|+\rangle_x$. Pick some arbitrary overall phase."
   - Relative & Overall Phase: Students may choose to represent the spin-1/2 state with various overall phases but pairs should have the same relative phase. It is important to point out to students that it's the relative phase that defines the state.

4. "Represent the state $e^{i\omega t}\,|+\rangle_x$. What does it look like? Does this state change with time?"
   - Relative & Overall Phase: This is a time-dependent overall phase, so both students should rotate their arms together preserving the relative phase.
   - Ask for a student to describe what they're doing in words.
   - The question is deliberately ambiguous. Although the vector changes with time, the quantum state does not change with time (i.e., measurement probabilities do not change)

5. "What kind of motion with your arms would result in a state that changes with time?"

- Relative & Overall Phase: Students need to move their arms at different rates and/or in different directions so that the relative phase changes.
- Same State Again: Students might notice that the time evolution is periodic - some time later, the state returns to the original state.

6. "How could you represent this time-evolving state in Dirac notation?"
   - $|\psi(t)\rangle = e^{i\omega_1\, t}c_+\,|+\rangle + e^{i\omega_2\, t}c_-\,|-\rangle$
   - **Optional:** "Is this vector normalized?" Demonstrate to students that adding time-dependent phases doesn't change the normalization of the state.

7. **Optional Concrete Example - Spin Precession**: Ask to student to start in the state $|+\rangle_x$ and act out the time evolution:

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}e^{i\omega\, t}\,|+\rangle + \frac{1}{\sqrt{2}}e^{-i\omega\, t}\,|-\rangle$$

   - Recognizing states: You can ask student to pause at key moments and try to recognize familiar states as they rotate: $|+\rangle_x$, $|-\rangle_x$, $|+\rangle_y$, and $|-\rangle_y$.

1.2 Wrap-up

The big take-home is that the relative phase must change with time in order for the state to change with time. Therefore, there must be a non-zero difference in the frequency of the phases on each term in the expansion.