Student handout

The Magnetic Vector Potential Due to a Spinning Ring of Charge

1. Use the superposition principle for the magnetic vector potential due to a continuous current distribution:

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}'(\vec{r}')}{|\vec{r}-\vec{r}'|}\, d\tau', \tag{1}$$

to find the magnetic vector potential everywhere in space due to a spinning charged ring with radius R, total charge Q, and period T.

2. Evaluate your expression for the special case that $\vec{r}$ is on the z-axis.

3. Evaluate your expression for the special case that $\vec{r}$ is on the x-axis.

4. Find a series expansion for the electrostatic potential at these special locations:

   a) Near the center of the ring, in the plane of the ring;
   b) Near the center of the ring, on the axis of the ring;
   c) Far from the ring on the axis of symmetry;
   d) Far from the ring, in the plane of the ring.

1 Instructor's Guide

1.1 Introduction

Students should be assigned to work in small groups given the following instructions using the visual of a hula hoop or other large ring:

Prompt: "This is a ring with radius R and total charge Q and spinning with period T. Find a formula for the magnetic vector potential $\vec{A}$ due to this ring that is valid everywhere in space".

1.2 Student Conversations

This activity is part of a sequence of four electrostatics activities involving a ring of charge: V, $\vec{E}$, $\vec{A}$, $\vec{B}$. They are arranged so that the mathematical complexity of the problems increases in a natural way. If you are doing this activity as a standalone, please see the Student Conversations section of the previous activities (Electrostatic Potential Due to a Ring of Charge, Electric Field Due to a Ring of Charge) for further advice.

Part I - Finding the potential everywhere in space

The new idea in the magnetic vector potential case is to find the linear current density (current) in the ring. Many students will have learned that current is "charge per time" or the derivative of charge with respect to time. Neither of these resources about current will be helpful to them here. They will need to know that current density is charge density times velocity:

$$\vec{I} = \lambda \vec{v} \tag{2}$$
$$= \frac{Q}{2\pi R}\frac{2\pi R}{T}\hat{\phi} \tag{3}$$

Be watchful. Many students will get the correct answer on dimensional grounds, but will not be able to justify their answer in a way that will extend to other problems.

Part II - Evaluate on the z-axis and on the x-axis

(See solution.)

Part III (Optional) - Series expansions

- With the charged ring in the $x, y-$plane, students will make the power series expansion for either near or far from the plane on the z axis or near or far from the z axis in the $x, y-$plane. Once all students have made significant progress toward finding the integral from part I, and some students have successfully determined it, then the instructor can quickly have a whole class discussion followed by telling students to now create a power series expansion. The instructor may choose to have the whole class do one particular case or have different groups do different cases.

- If you are doing this activity without having had students first create power series expansions for the electrostatic potential due to two charges, students will probably find this portion of the activity very challenging. If they have already done the Discrete Charges activity, or similar activity, students will probably be successful with the z axis case without a lot of assistance because it is very similar to the z axis case for the two $+Q$ point charges. However, the x axis presents a new challenges because the "something small" is two terms. It will probably not be obvious for students to let $\epsilon = \frac{2R}{s}\cos\phi' + \frac{R^2}{s^2}$ (see Eq. 17 in the solutions) and suggestions should be given to avoid having them stuck for a long period of time. Once this has been done, students may also have trouble combining terms of the same order. For example the ϵ^2 term results in a third and forth order term in the expansion and students may not realize that to get a valid third order expansion they need to calculate the ϵ^3 term.

1.3 Wrap-up

If you are doing this activity as a standalone, please see the Wrap-Up section of the previous activities (Electrostatic Potential Due to a Ring of Charge, Electric Field Due to a Ring of Charge) for further advice.