Student handout

The Electrostatic Field Due to a Ring of Charge

1. Use Coulomb's law
$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}')\,(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\,d\tau'$$
to find the electric field everywhere in space due to a charged ring with radius R and total charge Q.

2. Evaluate your expression for the special case that $\vec{r}$ is on the z-axis.

3. Evaluate your expression for the special case that $\vec{r}$ is on the x-axis.

4. Find a series expansion for the electric field at these special locations:

   a) Near the center of the ring, in the plane of the ring;
   b) Near the center of the ring, on the axis perpendicular to the plane of the ring;
   c) Far from the ring on the axis perpendicular to the plane of the ring;
   d) Far from the ring, in the plane of the ring;

1 Instructor's Guide

1.1 Introduction

Students should be assigned to work in small groups and given the following instructions using the visual of a hula hoop or other large ring:

Prompt: "This is a ring with radius R and total charge Q. Find a formula for the electric field $\vec{E}$ due to this ring that is valid everywhere in space".

1.2 Student Conversations

This activity is part of a sequence (the `https://paradigms.oregonstate.edu/sequences/3972/`) of four electrostatics activities involving a ring of charge: V, $\vec{E}$, $\vec{A}$, $\vec{B}$. They are arranged so that the mathematical complexity of the problems increases in a natural way. If you are doing this activity as a standalone, please see the Student Conversations section of the previous activity (Electrostatic Potential Due to a Ring of Charge) for further advice.

Part I - Finding the electric field everywhere in space

The new idea in the electric field case is that the numerator is a vector. The basis vectors in cylindrical or spherical coordinates differ from point to point in space. Therefore, you CANNOT subtract two vectors that "live" at different points if they are expanded in curvilinear coordinate basis vectors.

The numerator in this case must be expanded in rectangular basis vectors (so you can subtract) and components written in curvilinear coordinates (so that you can integrate)

$$\vec{r} - \vec{r}' = (s\cos\phi - R\cos\phi')\hat{x} + (s\sin\phi - R\sin\phi')\hat{y} + (z)\hat{z}$$

Part II (Optional) - Power series expansion along an axis
This part will go much the same as for the potential case.

1.3 Wrap-up

If you are doing this activity as a standalone, please see the Wrap-Up section of the previous activity (Electrostatic Potential Due to a Ring of Charge) for further advice.

New wrap-up content for this activity:

- Work through any details of the calculation that were problematic for groups, especially if the difficulties were not addressed in the groups.
- Show a graph of the value of the electric field superimposed on the electrostatic potential.
  - Compare to the electrostatic potential. Which is scalar and which is a vector? Where are these quantities large or small?
  - How are the shapes of these graphs predicted by the algebra?
  - How were the shapes of these graphs predicted by what the students sketched in activities: Drawing Equipotential Surfaces and Draw Vector Fields?
  - Show that the electric field is everywhere perpendicular to the electrostatic potential.
  - Show that the electric field vectors are large where the equipotential surfaces are close together.