Student handout

Power Series Coefficients

Consider the power series:

$$f(z) = \sum_{n=0}^{\infty} c_n (z - z_0)^n$$

expanded around the point z_0. The coefficients are found from the formula:

$$c_n = \frac{f^{(n)}(z_0)}{n!}$$

1. Find the first four non-zero coefficients for $\sin\theta$ expanded around the origin.

2. Write out the series approximation for $\sin\theta$ expanded around the origin including only the first four non-zero terms.

   $\sin\theta \approx$

3. Plot $\sin\theta$ and your series approximation term by term and notice where the approximation is good.

4. Find the 4th order coefficients for $\sin\theta$ expanded around $\theta_0 = \pi/6$.

5. Write out the series approximation, correct to 4th order, for $\sin\theta$ expanded around $\theta_0 = \pi/6$.

$\sin\theta \approx$

6. Plot $\sin\theta$ and your series approximation term by term and notice where the approximation is good.

7. What does it mean to write a series expansion around the point a?

8. Briefly describe in words how to expand the series approximation for a function, correct to 4th order.

1 Instructor's Guide

1.1 Introduction

Start with a brief lecture giving the form of power series expansions, introducing the basic vocabulary, and deriving the formula for the coefficients. See for example, the content in Power Series. Then pass out the handout, to be completed in small groups. It is important that all students get to through the first two questions. Fast groups can go on to the later questions.

1.2 Student Conversations

- Pay attention to the **name of the independent variable**. "The equation for the coefficients is given in terms of the variable z. What is the independent variable in $\sin\theta$?"
- Emphasize the difference between the **order** of a power series and the number of *nonzero terms*.
- Emphasize the difference between **expanding at** $\theta = \pi/6$, and replacing θ by $\theta - \pi/6$ in the series expansion at $\theta = 0$. If you are asked to find the power series expansion around $z = a$, then you must plug the number a into all of the derivatives.
- We find that once the coefficients are calculated, some students want to add the coefficients together rather than multiplying them by the appropriate monomial:

$$\cancel{f(z) = c_0 + c_1 + c_2 + ...}$$

Students should be encouraged to write out the general form of the expansion in terms of the coefficients:

$$f(z) = c_0 + c_1 (z - z_0) + c_2 (z - z_0)^2 + ...$$

- Some students don't understand the notation $f(z_0)$ as "evaluate the function at $z = z_0$".
- Some student aren't familiar with the convention that z_0 is a particular value of the variable z.

1.3 Wrap-up

In a whole class wrap-up, make sure to address all of the points in Student Conversations, above, especially the vocabulary. This activity is designed to be followed by the activity: Visualization of Power Series Approximations which let's students explore the graphs of the different orders of power series approximations that they have calculated in this activity.