Student handout

Calculating Total Charge

Each group will be given one of the charge distributions given below: (α and k are constants with dimensions appropriate for the specific example.)

- Spherical Symmetery
  1. A positively charged (dielectric) spherical shell of inner radius a and outer radius b with a spherically symmetric internal charge density $\rho(\vec{r}) = \alpha\, r^3$
  2. A positively charged (dielectric) spherical shell of inner radius a and outer radius b with a spherically symmetric internal charge density $\rho(\vec{r}) = \alpha\, e^{(kr)^3}$
  3. A positively charged (dielectric) spherical shell of inner radius a and outer radius b with a spherically symmetric internal charge density $\rho(\vec{r}) = \alpha\, \frac{1}{r^2}\, e^{(kr)}$
- Cylindrical Symmetry
  1. A positively charged (dielectric) cylindrical shell of inner radius a and outer radius b with a cylindrically symmetric internal charge density $\rho(\vec{r}) = \alpha\, s^3$
  2. A positively charged (dielectric) cylindrical shell of inner radius a and outer radius b with a cylindrically symmetric internal charge density $\rho(\vec{r}) = \alpha\, e^{(ks)^2}$
  3. A positively charged (dielectric) cylindrical shell of inner radius a and outer radius b with a cylindrically symmetric internal charge density $\rho(\vec{r}) = \alpha\, \frac{1}{s}\, e^{(ks)}$

For your group's case, answer the following questions:

1. Find the total charge. (If the total charge is infinite, decide what you should calculate instead to provide a meaningful answer.)
2. Find the dimensions of the constants α and k.

1 Instructor's Guide

1.1 Introduction

We usually start with a mini-lecture reminder that total charge is calculated by integrating over the charge density by chopping up the charge density, multiplying by the appropriate geometric differential (length, area, or volume element), and adding up the contribution from each of the pieces. **Chop, Multiply, Add** is a mantra that we want students to use whenever they are doing integration in a physical context.

The students should already know formulas for the volume elements in cylindrical and spherical coordinates. We recommend Scalar Surface and Volume Elements as a prerequisite.

We start the activity with the formulas $Q = \int \rho(\vec{r}')d\tau'$, $Q = \int \sigma(\vec{r}')dA'$, and $Q = \int \lambda(\vec{r}')ds'$ written on the board. We emphasize that choosing the appropriate formula by looking at the geometry of the problem they are doing, is part of the task.

Each student group is assigned a particular charge density that varies in space and asked to calculate the total charge. This activity is an example of `https://paradigms.oregonstate.edu/whitepaper/compare-and-contrast-activity`.

1.2 Student Conversations

This activity helps students practice the mechanics of making total charge calculations.

- **Order of Integration** When doing multiple integrals, students rarely think about the geometric interpretation of the order of integration. If they do the r integral first, then they are integrating along a radial line. What about θ and ϕ. If this topic does not come up in the small groups, it makes a rich discussion in the wrap-up.
- **Limits of Integration** some students need some practice determining the limits of the integrals. This issue becomes especially important for the groups working with a cylinder - the handout does not give the students a height of the cylinder. There are two acceptable resolutions to this situation. Students can "name the thing they don't know" and leave the height as a parameter of the problem. Students can also give the answer as the total charge per unit length. We usually talk the groups through both of these options.
- **Dimensions** Students have some trouble determining the dimensions of constants. Making students talk through their reasoning is an excellent exercise. In particular, they should know that the argument of the exponential function (indeed, the argument of any special fuction other than the logarithm) must be dimensionless.
- **Integration** Some students need a refresher in integrating exponentials and making u-substitutions.

1.3 Wrap-up

You might ask two groups to present their solutions, one spherical and one cylindrical so that everyone can see an example of both. Examples (b) and (f) are nice illustrative examples.