Student handout Each small group is assigned a spherical harmonic from the list below:

1. Y_1^1
2. Y_1^0
3. Y_1^{-1}
4. Y_2^1
5. Y_2^0
6. Y_2^{-1}

Using a tiny Argand diagram, represent the value of the spherical harmonic at:

- at the equator ($\theta = \pi/2$) for $\phi = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$
- repeat for $\theta = \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}$

Tip: Make reference marks in black and draw the complex value of the spherical harmonic in a different color.

1 Instructor's Guide

1.1 Main Ideas

- Spherical harmonics are continuous functions on the surface of a sphere.
- Spherical harmonics are complex valued functions.
- The ℓ and m values tell us how the function oscillates across the surface.
  - $l =$ the total number of nodal circles on the sphere
  - $|m| =$ the number of nodal circles along longitudes (or the number of 2π oscillations in the complex plane around lines of latitude; the direction tells you which direction the phase changes)
  - $l - |m| =$ number of nodal circles along lattitudes

1.2 Students' Task

Students set up a spherical coordinate system on a balloon, draw a spherical harmonic, and use the balloon as a prop to describe the main features of their spherical harmonic to the class.

1.3 Prerequisite Knowledge

Students should be familiar with the spherical coordinate system, and the equations describing spherical harmonics.

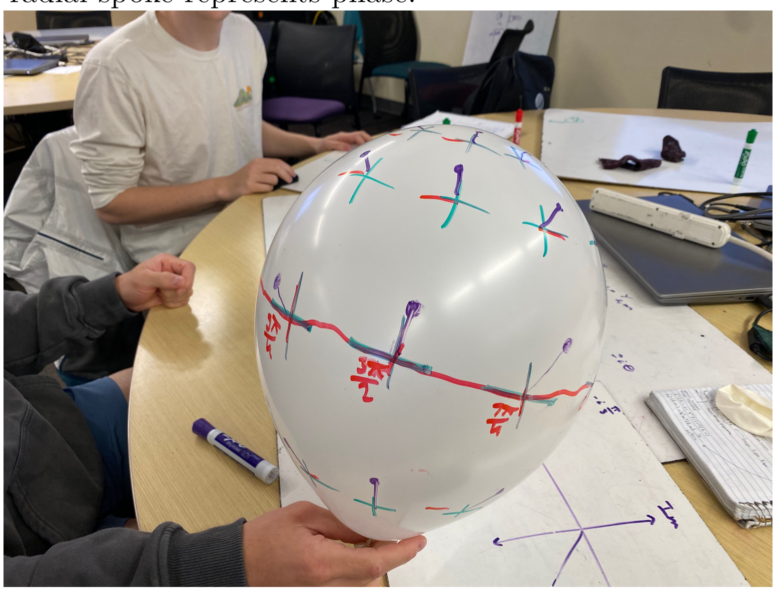
1.4 Props/Equipment

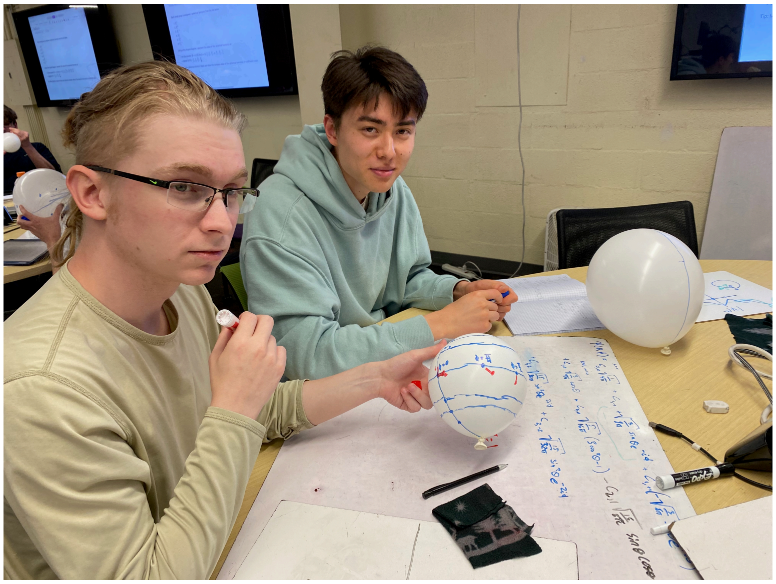
- Punch ball balloons (almost spherical), or standard party balloons (less perfect).
- White board markers

1.5 Activity: Introduction

The instructor points out that visualizing complex numbers on a spherical surface is challenging, and then describes a way to visually represent a field of complex numbers with a set of Argand diagrams (examples below).

Note that the size of each circle on the Argand diagram represents magnitude, and the direction of the radial spoke represents phase.





1.6 Activity: Student Conversations

During the activity the teachhing team should ask questions such as

- Show me $\phi = 0$ on your sphere
- Show me $\theta = \pi/2$ on your sphere
- Is your function well behaved (no discontinuities) everywhere on the surface?
- If students need help with their coordinate system, ask them to draw a dot representing $\theta = 0$ and line to represent all points with $\phi = 0$.
- Not all students fully understand the stylized Argand diagrams, but most can still use them appropriately.

1.7 Activity: Wrap-up

After students have made their plots, each group shows the main features of their spherical harmonic. For example,

- Are there nodes at the poles?
  - *Unless $m = 0$, the spherical harmonics must have nodes (zeros) at the $\theta = 0$ and $\theta = \pi$ to ensure a continuous function.*
- How does the phase change moving around the equator?
  - *The index m is equivalent to the number of times the phase gets wound as you move around the equator.*

- How is $m = 1$ different from $m = -1$?
- What happens if we add two spherical harmonic functions? For example ($\ell = 1$, $m = 1$) and ($\ell = 1$ $m = -1$).
  - *Discuss the constructive and destructive interference of complex numbers.*

1.8 Extensions

- What linear combination of the $\ell = 0$ and $\ell = 1$ spherical harmonics would describe a probability density that is big in the northern hemisphere and small in the southern hemisphere?
  - *Combinations of Y_0^0 and Y_1^0, each having the same sign. $Y_1^{\pm 1}$ are symmatric across the xy-plane and wouldn't yield north/south assymmetries.*
- What linear combination of the $\ell = 0$ and $\ell = 1$ spherical harmonics would describe a probability density that is big in the $x$direction but and small in $-x$ direction?
  - *Combinations of Y_1^{-1} - Y_1^1. A equal difference of these harmonics will yield a $\cos\phi$ pattern which is large along the x axis.*

1.9 Alternate Activity

You can shorten the activity by only asking the students to show how the phase (not the magnitude) changes about the equator. Have one group at each table show $e^{i\phi}$ and one group show $e^{-i\phi}$ around the equator. By asking them to then add their functions together, you can use this as a way of introducing the superposition of states and to talk about how physicists' counting of states (p_1, p_0, p_{-1}) differs from chemists' counting of states (p_x, p_y, p_z) (Chemists use linear combinations of the complex spherical harmonics that are pure real).

$$p_x = \frac{1}{\sqrt{2}}\left(Y_1^{-1} - Y_1^1\right)$$
$$p_y = \frac{i}{\sqrt{2}}\left(Y_1^{-1} + Y_1^1\right)$$
$$p_z = Y_1^0$$