Student handout In the Mathematica Worksheet Conics.nb or the online Geogebra visualization at GMM: Graphs in Polar Coordinates, you will examine a three parameter family of curves described by the polar equation

$$r(\phi) = \frac{\alpha}{1 + \epsilon\cos(\phi + \delta)}.$$

Describe in detail how the shape of the plot depends on the parameters α, δ, and ϵ. Pay particular attention to different values of ϵ.

1 Instructor's Guide

1.1 Introduction

This activity was originally designed to provide students an opportunity to explore polar plots of conic sections in a pure math environment, so that when they see the derivation of the formula for orbital motion, they will immediately recognize it as the polar formula for a conic section. In this implementation, you can do the activity any time before that derivation. Make sure to include an opportunity (in-class or homework) for the students to work out the relationship between the mathematical parameters (ellipticity, etc.) and the physical ones (angular momentum, etc.)

Other faculty have chosen to use this activity after a lecture derivation of the equations of motion the two-body central force problem

$$r(\theta) = \frac{\frac{l^2}{\mu k}}{1 + C' \cos{(\phi + \delta)}}.$$

A discussion of polar plots (how they are generated, how they are different from the usual cartesian plots) takes place just before students are released to play with the Mathematica notebook or Geogebra applet. This latter order offers less opportunity for students to discover things for themselves.

1.2 Student Conversations

- The students are asked to determine how the constants α, ϵ, and δ change the shape of the orbit.
- Students should be encouraged to identify the geometric shape of the resulting conic section and recall what they know about those shapes. (i.e. the Cartesian equation for each shape, its special properties, etc.)
- Students often discuss real world orbiting bodies in the context of this activity. One common confusion is whether comets have elliptic, parabolic,or hyperbolic orbits. This is a good opportunity to get them to recall what they know about comets (they typically appear periodically) with what they know about different types of orbits. After some discussion, students should be able to conclude that comets must have highly elliptical orbits rather than hyperbolic or parabolic orbits.

1.3 Wrap-up

- It is useful to spend a few minutes reviewing what the students observed about the behavior of the variables:
  - α adjusts the scale of the orbit
  - ϵ, the eccentricity, adjusts the shape of the orbit.

$\epsilon = 0$	circle
$0 < \epsilon < 1$	ellipse
$\epsilon = 1$	parabola
$\epsilon > 1$	hyperbola

  - δ the phase shift adjusts the rotation of the major axis of the conic.
- It is useful to get students to look at the extreme cases of the conic equation and examine what the maximum value of r will be for $\epsilon = 0$, $\epsilon < 1$, $\epsilon = 1$, $\epsilon > 1$
- To shorten the activity, you can ask about α and ϵ only (not δ).

1.4 Extensions

- It is important to make sure students make the connection between the variables in the worksheet and those in the derivation for the shape of the orbit. $\alpha = \frac{l^2}{\mu k}$ and $\epsilon = C'$. This can happen during the activity or later, depending on the timing of the lecture about orbits.