Student handout Download and run this *Mathematica* notebook or this Geogebra applet. You have four different sliders that control the values of four parameters k, ℓ, μ, and E. Answer the following questions:

1. What is the physical/geometric meaning of each parameter k, ℓ, μ, E?
2. How does each parameter k, ℓ, μ, E affect the plot?
3. Which term in the effective potential $\left(-\frac{k}{r} \text{ or } \frac{\ell^2}{2\mu r^2}\right)$ dominates for small values of r? For large values of r? Explain in terms of both the equation and the graph.
4. What are the classical turning points? Under what conditions will the particle be bound? Unbound?
5. How do your answers for the last question change (if at all) if you consider a repulsive potential? Hint: Figure out what you must change in this notebook and investigate.

1 Instructor's Guide

1.1 Prerequisite Knowledge

- Students should know that the effective potential is used to reduce the 2-D central force problem to a 1-D problem.
- Students should know that the shape of the orbit need not be elliptical - all conic sections are solutions to the equations of motion (depending on the values of the various parameters).

1.2 Introduction

We usually start this activity with a general discussion about sets of solutions to an equation and the professional sense-making activity of exploring parameter space to build an understanding of the types of solutions that exist in the set.

1.3 Student Conversations

- In exploring the parameter space of solutions, students should be encouraged to identify limiting and special cases (e.g. when is the orbit is circular, what is the shape of the function for large and small separations, etc).
- Some students point out (correctly) that the force constant k ought to depend on the reduced mass μ. This is a result of the gravitational force being the typical example of a central force; reminding students that other central forces exist (e.g, the Coulomb force) clears up any concerns.

- Some students will struggle with how the effective potential diagram is representing an orbit since it is supressing the 2D nature of the orbit. This activity works in conjunction with activity 740 which should help them discover how these pictures fit together.