Student handout For the state

$$|\Psi\rangle = \sqrt{\frac{7}{10}}|2,1,0\rangle + \sqrt{\frac{1}{10}}|3,2,1\rangle + i\sqrt{\frac{2}{10}}|3,1,1\rangle$$

Calculate

- $\mathcal{P}(L_z = \hbar)$
- $\langle L_z \rangle$
  Then, if you have time, continue with these calculations:
- $\mathcal{P}(L^2 = 2\hbar^2)$
- $\langle L^2 \rangle$
- $\mathcal{P}(E = -13.6eV/3^2 = -1.51eV)$
- $\langle E \rangle$
- What measurements can be degenerate on the Hydrogen atom?
- What is the time development of this state?
- What is the probability of finding the particle in the region $0 < \theta < \pi/6$, $\pi/3 < \phi < \pi/2$, and $r_1 < r < r_2$?

1 Instructor's Guide

1.1 Students' Task

Students are asked to find eigenvalues, probabilities, and expectation values for H, L^2, and L_z for a superposition of $|n\ell m\rangle$ states in ket notation. This can be done on small whiteboards or with the students working in groups on large whiteboards.

1.2 Introduction

Write a linear combination of $|nlm\rangle$ states on the board. For example:

$$\Psi = \sqrt{\frac{7}{10}}|2,1,0\rangle + \sqrt{\frac{1}{10}}|3,2,1\rangle + i\sqrt{\frac{2}{10}}|3,1,1\rangle$$

(it is a good idea to provide a state that is degenerate in one or more of the quantum numbers).

Then ask the students a series of small whiteboard questions with a short wrap-up after each one that reiterates key points (see wrap-up below).

- $\mathcal{P}_{L_z=m\hbar}$;
- $\langle L_z \rangle$
- $\mathcal{P}_{L^2=\ell(\ell+1)\hbar^2}$;
- $\langle L^2 \rangle$
- $\mathcal{P}_{E=-13.6eV/n^2}$;
- $\langle E \rangle$

1.3 Student Conversations

- Students can often find probabilities by inspection at this point, so it is sometimes helpful to ask them to write out explicitly what they did.
- Discuss explicitly how degeneracy is handled by summing over the relevant probabilities.
- Not every student will make it to the last question but it should be a focus of the wrap up and the conversation about summation over quantum numbers when taking probabilties or expecation values.

1.4 Wrap-up

- Summarize explicitly how to calculate the probability of degenerate states by summing the probabilities.
- It is a good idea to do the wrap-up after each small whiteboard question to help to keep the whole class together.
- Many students want to avoid explicitly putting the summation in, so this is a good time to talk explicitly about the summation limits for each case. For example, in finding the probability for measuring L_z to be $-1\hbar$, students want to simply write:

  $$\mathcal{P}_{L_z=-\hbar} = |\langle n, \ell, -1|\Psi\rangle|^2$$

  instead of

  $$\mathcal{P}_{L_z=-\hbar} = \sum_{n=2}^{\infty} \sum_{\ell=1}^{2} |\langle n, \ell, -1|\Psi\rangle|^2$$

- This is a good time to also address the difference between the sum of squares and the square of sums, which is an ongoing issue for many students.
- The end is the moment to ask the last question to the whole class and come to the realization that all three of our discrete operators can be degernerate on the hydrogen atom and that they are all now distinct in terms of quantum numbers.

1.5 Extension

The same state is used to explore matrix representations on the hydrogen atom in Hydrogen Probabilities in Matrix Notation