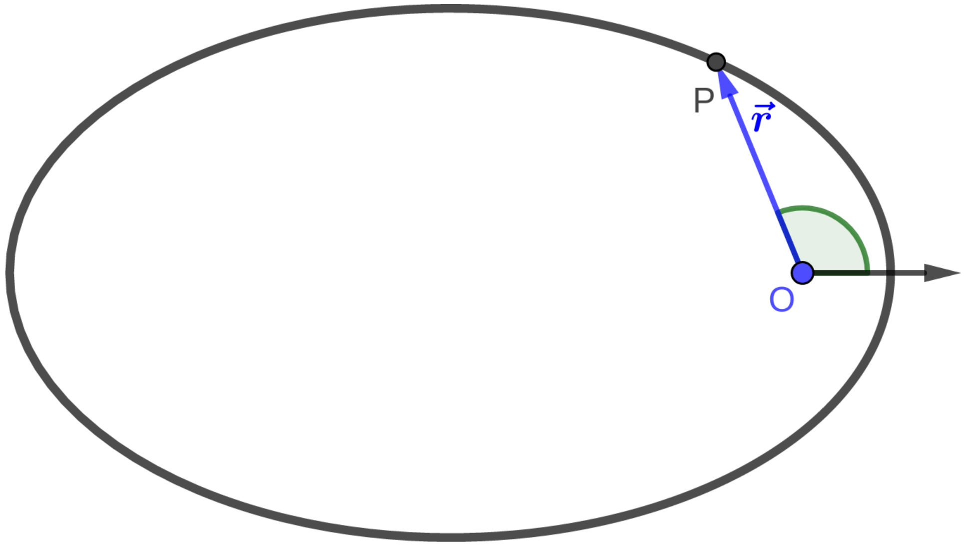
Student handout

1. On the figure below, draw $\hat{s}$ and $\hat{\phi}$ at P.



2. Find $\frac{d}{dt}\hat{s}$ and $\frac{d}{dt}\hat{\phi}$ in terms of $\hat{s}$ and $\hat{\phi}$.

3. Find $\vec{v}$ in terms of $\hat{s}$ and $\hat{\phi}$.

1 Instructor's Guide

1.1 Introduction

The activity begins by with a SWBQ asking the students to find $\vec{v} = \frac{d\vec{r}}{dt}$. Typically, students propose two alternatives,

$$\frac{d\vec{r}}{dt} = \frac{ds}{dt}\hat{s}$$
$$\frac{d\vec{r}}{dt} = \frac{ds}{dt}\hat{s} + \frac{d\phi}{dt}\hat{\phi}.$$

A discussion ensues about which is correct. The proper formula is derived using

$$\begin{aligned}\frac{d\vec{r}}{dt} &= \frac{d}{dt}(s\hat{s}) \\ &= \frac{ds}{dt}\hat{s} + s\frac{d\hat{s}}{dt}.\end{aligned}$$

This result is further justified by drawing the velocity vector and discussing the fact that $\hat{r}$ changes as ϕ changes.

Students are then asked to find values for $\frac{d\hat{\mathbf{s}}}{dt}$ and $\frac{d\hat{\phi}}{dt}$. Students are given the attached handout and asked to carry out calculations of $\vec{v}$ and $\vec{a}$ in polar coordinates.

1.2 Student Conversations

- The most straightforward way to find the time dependence of $\hat{s}$ and $\hat{\phi}$ is to write them in terms of $\hat{x}$ and $\hat{y}$. Since $\hat{x}$ and $\hat{y}$ to not vary with position and therefore do not vary with time (as the particle moves around), their time derivatives are zero. All of the time dependence is in the coefficients.

- Most students will not realize that the derivatives of $\hat{s}$ and $\hat{\phi}$ can be rewritten in terms of $\hat{s}$ and $\hat{\phi}$. You will need to check all the groups and prompt this, where necessary. Interrupt the activity for a mini-lecture, if necessary.

- Students may have trouble knowing when and how to apply the product rule and the chain rule. Most students will omit the factors of $\dot{\phi}$.

- One common student confusion is to believe that since there is no explicit time dependence in the equation $\vec{r} = s\hat{s}$, then there is no time dependence and $\frac{d\vec{r}}{dt} = 0$. This provides the teacher an opportunity to remind the students that
  - To save time and writing, the time dependence of a variable is often not stated explicitly;
  - It is important to think carefully about what variables we think will vary with time and only set their time dependence equal to zero if we have a good reason.

- Students often forget that $\hat{s}$ and $\hat{\phi}$ vary with ϕ. It is useful to point out that Cartesian coordinates and polar coordinates are different because the unit vectors in Cartesian coordinates have fixed directions but the directions of polar coordinates are not fixed. It may be useful to emphasize at this time that they will encounter unit vectors whose direction is dependent on the position coordinates often in this and future physics classes.

- If you haven't talked about it before, sometimes students will want to pattern match from $\vec{r} = x\hat{x} + y\hat{y}$ to $\vec{r} = s\hat{s} + \phi\hat{\phi}$. Some of them may realize that the second term has the wrong dimensions and add in s's, so it is important to have them take a step back and think about how to describe a position in polar coordinates.

- Depending on the background of your students, this can be a very time consuming activity. Sometimes it is best to start the activity in class so that you can help students get started, but then to have them finish the calculations at home.

1.3 Wrap-up

After students have completed the calculations, quickly review the answers and the procedure for calculating them.

$$\begin{aligned}\vec{v} &= \dot{\vec{r}} \\ &= \dot{r}\hat{s} + s\dot{\phi}\hat{\phi} \\ \vec{a} &= \dot{\vec{v}} \\ &= \ddot{\vec{r}} \\ &= \left(\ddot{s} - s\dot{\phi}^2\right)\hat{s} + \left(s\ddot{\phi} + 2\dot{s}\dot{\phi}\right)\hat{\phi}\end{aligned}$$

Notes on these derivations can be found in these lecture notes.

Also, ask students to look at the dimensions. Each term has two derivatives with respect to t and one factor of r or its derivatives.

1.4 Extensions

This activity leads smoothly into a mini-lecture defining the kinetic energy and angular momentum in polar coordinates. If you have a group (or 2) that are ahead of the game, ask them to find $\vec{L}$ in terms of polar coordinates and have them start the mini-lecture.