Student handout

Use this *Mathematica* worksheet to explore the spatial properties of linear combinations of spherical harmonics $Y_\ell^m(\theta, \phi)$.

1 Instructor's Guide

1.1 Introduction

The activity is introduced by reminding students that any function on the sphere can be written as a linear combination of the Spherical Harmonics, since they form an orthogonal basis for the space of the sphere. This worksheet plots the square of the norm of the function (probability density in quantum mechanics). Students are also reminded that the probability density is represented by the color in the case of the first sphere plot, that the polar plot (the second to last one in the worksheet) indicates the value by both the color and the distance from the origin and the final graph indicates the value by both the color and the distance from the sphere. It is important to caution the students that this worksheet only shows the angular part and that these functions do not contain any information about the radial dependence of the hydrogen atom wavefunctions.

1.2 Student Conversations

- A good question to help frame students exploration is to ask them to identify how the combinations of $Y_{\ell,m}$s are different from individual $Y_{\ell,m}$s. In particular, students will notice that the axial symmetry common to all of the individual $Y_{\ell,m}$s is not present for all combinations. This may seem counter-intuitive to them and leads to a good discussion of the role of the complex phase in the ϕ part of the spherical harmonics.

- Some particularly interesting states to recommend students view are included. Note the role of the relative phase.

1.3 Wrap-up

- It is useful to get students to draw some conclusions about when you will and when you will not see axial symmetry with combinations of spherical harmonics.