> **Student handout** On your whiteboard, there should be a 5x5 square grid of dots. The instructor will draw a specific vector $\vec{k}$ on your grid.
>
> For your $\vec{k}$, connect dots with the same value of $\vec{k} \cdot \vec{r}$.

1 Instructor's Guide

1.1 Students' Task

Each small group of 3-4 students is given a white board or piece of paper with a square grid of points on it.

Each group is given a different two-dimensional vector $\vec{k}$ and is asked to calculate the value of $\vec{k} \cdot \vec{r}$ for each point on the grid and to draw the set of points with constant value of $\vec{k} \cdot \vec{r}$ using rainbow colors to indicate increasing value.

1.2 Prerequisites

Students should know both the rectangular component and the geometric definitions of the dot product. Some of this activity can only be used if students know a little about complex number algebra, including Euler's formula.

1.3 Student Conversations

The group part of this activity should be quite quick, 5-10 minutes.

- Go around to each group and mark an origin on the grid and a vector from the origin $\vec{k}$. Choose simple vectors, not longer than a few grid points. Choose a variety of vectors so that the compare and contrast wrap-up will work well.
- Some groups will have trouble getting started, particularly if they try to use the geometric definition of the dot product. Ask them if they know anything else about the dot product. This activity works well if preceded by Dot Product Review which prompts students to think about various definitions and representations of the dot product.
- Some groups will have trouble understanding what is meant by the position vector. Do not leave them stuck here. It is usually sufficient to draw one. If necessary, ask what the components are of the vector you have drawn.
- Prompt the students to connect the points with constant value of $\vec{k} \cdot \vec{r}$. Remind them to color code these connections with rainbow colors. Don't give away the punchline by asking the to draw the "lines" of constant $\vec{k} \cdot \vec{r}$.

1.4 Wrap-up

This is a compare and contrast activity. Ask each group to present. They should show their white board, show their vector $\vec{k}$, and their curves of constant k.

Points that should arise:

- The points of constant $\vec{k} \cdot \vec{r}$ are straight lines.
- These lines are perpendicular to $\vec{k}$.
- The spacing between the lines is smaller when $\vec{k}$ is longer.

Questions to ask students:

1. **Why are the lines of constant $\vec{k} \cdot \vec{r}$ perpendicular to $\vec{k}$?**

   Usually someone in the class can come up with the explanation that all the position vectors $\vec{r}$ that have the same projection onto $\vec{k}$ have the same value of $\vec{k} \cdot \vec{r}$.

   This projection is easiest to interpret when I think about the dot product as $\vec{k} \cdot \vec{r} = |\vec{k}||\vec{r}| \cos\theta$, where $|\vec{r}| \cos\theta$ is the projection of $\vec{r}$ onto $\vec{k}$.

   This is a good time to remind the students that they have had to use two different representations of the dot product to completely understand this problem.

2. **What if the grid of points was made a 3D cube of points? What would constant $\vec{k} \cdot \vec{r}$ look like?**

   Constant $\vec{k} \cdot \vec{r}$ are planes.

3. **What does $\cos(\vec{k} \cdot \vec{r})$ look like?**

   Planes that vary in value from -1 to 1 sinusoidally.

4. **What does $E_0 \cos(\vec{k} \cdot \vec{r})$ look like?**

   Planes that vary in value from -E_0 to E_0 sinusoidally.

5. **What does $E_0 \cos(\vec{k} \cdot \vec{r} - \omega t)$ look like?**

   Planes that move in the direction of $\vec{k}$ at a rate of ω/k.