Student handout

1. **Measurement**

   a) Find the rate of change in the surface in the x-direction at the **blue** dot on your surface. Include units.

   $$\frac{\partial f}{\partial x} = \underline{\hspace{5cm}}$$

   b) Find the rate of change in the surface in the y-direction at the **blue** dot on your surface. Include units.

   $$\frac{\partial f}{\partial y} = \underline{\hspace{5cm}}$$

   c) Draw an arbitrary vector $\vec{\boldsymbol{u}}$ at the blue dot *on the contour mat*. What are its components?

   $$\vec{\boldsymbol{u}} = \underline{\hspace{5cm}}$$

   d) Find the rate of change in the surface in the $\vec{\boldsymbol{u}}$-direction. Include units.

   $$\frac{df}{ds} = \underline{\hspace{5cm}}$$

2. **Computation**

   a) Determine the gradient of f at the blue dot.

   $$\vec{\boldsymbol{\nabla}} f = \underline{\hspace{5cm}}$$

   b) Use the Master Formula to express $\frac{df}{ds}$ in terms of $\vec{\boldsymbol{\nabla}} f$, and compute the result.

   $$\frac{df}{ds} = \underline{\hspace{5cm}}$$

3. **Comparison**

   - Compare your answers.

1 Instructor's Guide

1.1 Main Ideas

- Measuring slope along various directions at a point on the surface

- Experimentally determining the gradient at a point on the surface
- Using the *Master Formula*

1.2 Prerequisite Knowledge

Students should be able to:

- Use the measurement tool to approximate a derivative from a plastic surface

1.3 Props/Equipment

- Tabletop Whiteboard with markers
- Plastic surface with contour map and inclinometer
- A handout for each student

1.4 Student Conversations

- Some students may believe that vectors are tied to the coordinate axes, and not the contour maps; that moving the coordinate axes moves the vector along with them so that no matter the direction of the coordinate axes, the vector is always pointing in the same direction relative to those axes.
- Some students may believe that the gradient will always point towards the top of the hill.
- Some groups may make choose arbitrary vectors that trivialize the calculations in this activity.
- Some students may believe that the gradient is a vector's property and not a point's property; that every vector at a given point has its own, but not necessarily unique, gradient. These students may justify their reasoning by saying that the steepest direction for any given vector must be along that vector.
- Some students may believe that longer vectors always have a greater slope / rate of change than shorter vectors, or that vector length is what denotes magnitude of slope / rate of change; that a vector's rate of change and its magnitude represent the same value.
- Some students may believe that every vector pointing in the same direction as the gradient is identical to the gradient vector.
- Some students may believe that a short vector on the contour map will always correspond to a short vector on the surface.
- Some students may believe that taking the dot product of two vectors produces a vector.