Student handout **Warm-up:** Imagine you are standing on the side of a tall hill. List three things you would want to know about your position.

On your Mark: Place your surface on the grid. Label the x and y directions on the grid and surface. Measure the slope *in the direction of greatest increase* of the surface at the blue dot. Include units.

Slope in steepest direction: ________________________

Get Set: The surface's height h is a function of x and y, giving $h = h(x, y)$. At the blue dot, measure both $\frac{\partial h}{\partial x}$ and $\frac{\partial h}{\partial y}$. Then form the vector $\frac{\partial h}{\partial x}\hat{\boldsymbol{x}} + \frac{\partial h}{\partial y}\hat{\boldsymbol{y}}$. Include units.

$\frac{\partial h}{\partial x} =$ _________ $\frac{\partial h}{\partial y} =$ _________ $\frac{\partial h}{\partial x}\hat{\boldsymbol{x}} + \frac{\partial h}{\partial y}\hat{\boldsymbol{y}} =$ _________ $\hat{\boldsymbol{x}} +$ _________ $\hat{\boldsymbol{y}}$

Go: At the blue dot, which way does your vector $\frac{\partial h}{\partial x}\hat{\boldsymbol{x}} + \frac{\partial h}{\partial y}\hat{\boldsymbol{y}}$ point on the surface?

1. What is your vector's magnitude?
2. How does your vector relate to the level curve through the blue dot?

Challenge: Rotate the surface 30° on the grid and redraw the x and y directions on your surface. Which of your answers to **On your Mark**, **Get Set**, and **Go** remain the same?

1 Instructor's Guide

1.1 Main Ideas

- The gradient is a geometric representation of the steepest slope and direction.

1.2 Students' Task

Students work in groups to measure the steepest slope and direction, and to compare their result with the gradient vector, obtained by measuring its components (the slopes in the coordinate directions).

1.3 Prerequisite Knowledge

- Familiarity with measuring partial derivatives using surfaces

- Basic knowledge of vectors

1.4 Introduction

No introduction is needed, provided the prerequisite content (partial derivatives and vectors) has been covered. In particular, it is *not* necessary to have discussed the gradient yet.

1.5 Student Conversations

Encourage students to measure their slopes as accurately as possible. (The correct slope should be approximately 1.16 in each case!)

Some groups will forget to keep track of signs, especially if the slopes in the x- and y-directions are different.

Ask each group to draw their gradient vector on their contour map. This allows the instructor to quickly determine whether their answers are correct, as they gradient should be perpendicular to the level curves.

Some students may need to be prodded to draw these vectors in the right place on the contour maps, as some students may use the contour maps merely as a convenient writing surface, without correctly aligning their surface with the map. Ask such students where the blue dot is *on the map*.

Be *very* careful as to not lead students to believe that *all* vectors' magnitudes depend on their slopes. This misunderstanding was common in a later activity.

Students may not understand why the gradient vector is perpendicular to the level curve passing through the point in question.

1.6 Wrap-up

Ask students to present their final drawing, showing the gradient vector on the map. (A document camera can be useful for this purpose.) Most students will quickly realize that their vector was indeed perpendicular to the level curves, and some should also realize that their computed magnitude (based on the partial derivatives in the coordinate directions) agrees reasonably well with their measured slope in the steepest direction.

1.7 Extensions

A natural followup activity is The Hill. A part of this latter activity is to emphasize that the gradient vector in these examples is in fact horizontal; it has no $\hat{\boldsymbol{z}}$-component. (This fact can also be brought out in the wrap-up, without using the Hill activity.)

Similarly, the Hill activity emphasizes that the gradient vector does *not* point toward the top of the hill, which could also be brought out during the wrap-up.

Both of these features activity can be brought out by asking students to stand up, imagining themselves standing on the contour diagram for the Hill activity, close their eyes, and stick their right hand out in the direction of the gradient vector at their location. (The instructor should designate one corner of the room to represent the center of the diagram.) Many students will incorrectly point "up", while some may incorrectly point to the top.