Student handout

Commutation Relations for Spin Operators

A commutator of two observables is defined as:

$$[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$$

Determine the results of the following commutators:

1. $[\hat{S}_x, \hat{S}_y]$
2. $[\hat{S}_y, \hat{S}_z]$
3. $[\hat{S}_z, \hat{S}_x]$
4. $[\hat{S}_y, \hat{S}_x]$
5. $[\hat{S}_z, \hat{S}_y]$
6. $[\hat{S}_x, \hat{S}_z]$

Remember that the matrix representation of the spin operators written in the S_z basis is:

$$\hat{S}_x \doteq \frac{\hbar}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \hat{S}_y \doteq \frac{\hbar}{2}\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad \hat{S}_z \doteq \frac{\hbar}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

1 Activity: Introduction

Divide students into groups to work out whether the spin operators commute.

2 Activity: Wrap-up

Groups should find that none of the quantum operators commute and therefore do not share the same basis for their respective eigenvectors. Because of this, it provides mathematical evidence for many properties that have so far been only observed. Since none of them commute, none of them have the same basis, nor can the spin operators be measured simultaneously.