Student handout For each of the vector fields below, decide whether the z-component of the curl is postive, negative, or zero in each quadrant. Be prepared to defend your answers.

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1 Instructor's Guide

1.1 Introduction

We precede this activity with a derivation of the rectangular coordinate expression for curl: (the magnitude of a particular component of) the curl is the circulation per unit area around an appropriately chosen planar loop. Our derivation follows the one in *Div, grad, curl and all that*, Schey, 2nd edition, Norton, 1973, p. 74.

Then the students are presented with a number of examples of 2-d vector fields in dry eraseable plastic sleeves. Most vector fields are shown as a cross-section of the field and it is assumed the the vector field is independent of the third (unshown) dimension. Students are asked to use the definition of curl to predict the sign and relative magnitude of the curl in each quadrant. Optionally, a prepared *Mathematica* worksheet can be used to calculate the curl, so students can check their predictions.

1.2 Student Conversations

- **Symmetry:** Students should be encouraged to see that it is easier to choose a loop that respects the symmetries of the vector field, i.e. pineapple chunks for cylindrical fields, etc.
- **Various Points:** Make sure to look at several different points in space for each vector field, not just the origin. Use the result to emphasize that curl is itself a field.
- **Positive or negative curl:** Students should should see that, for the two vector fields that circle the origin, different length scalings on the two fields lead to different signs for the curl, depending on whether they are adding larger vectors along the longer length arc or smaller vectors along the longer length arc. The last example $\left(\frac{1}{r}\hat{\phi}\right)$ can be framed as a "Jeopardy question" where students are asked to discover which scaling leads to zero curl everywhere except at the origin.

    1. The ANSWER is: a nontrivial field that looks like the one on the screen which has zero curl everywhere but the origin.
    2. What is the QUESTION? What is the magnetic field around a current carrying wire? Nature picks out this special case.

3. It is subtle (with the Delta function curl!) and surprising for students, so it is often worth talking/working through the origin and non-origin cases separately for the vanishing of a curl that looks curly nearly everywhere.

1.3 Wrap-up

A quick whole class discussion of the items listed in Student Conversations.