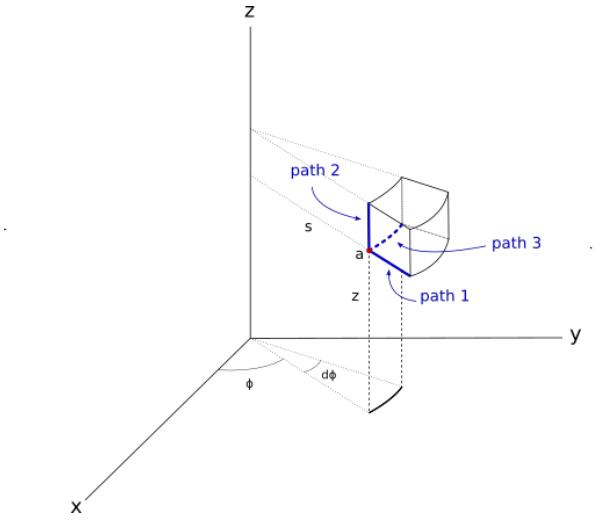
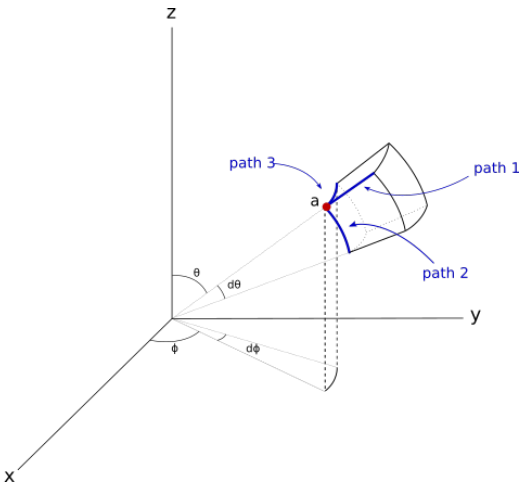
Student handout

1. Find the formulas for the differential area elements for:
   a) a plane (choose a easy one)
   b) a finite cylindrical shell (to the top/bottom and side separately),
   c) a spherical shell.
2. Find the formulas for the differential volume elements for:
   a) a thick slab
   b) a cylinder
   c) a sphere





1 Instructor's Guide

1.1 Notes:

This activity is identical to Scalar Surface and Volume Elements except uses a more sophisticated vector approach to find directed surface, and volume elements.

This activity can be done simultaneously with Pineapples and Pumpkins where students or the instructor cut volume elements out of pineapples and/or pumpkins.

1.2 Introduction

In a previous activity, Vector Differential–Curvilinear, students are asked to find the vector line element, $d\vec{\boldsymbol{r}}$, along each side of an "infinitesimal box" in cylindrical and spherical coordinates. Using the $d\vec{\boldsymbol{r}}$, they

are now asked to construct the area ($d\vec{\boldsymbol{A}}$) and volume (dV) elements in each coordinate system. This prepares students to integrate vector- and scalar-valued functions in curvilinear coordinates.

Begin with a brief lecture which "derives" the formula

$$d\vec{\boldsymbol{A}} = d\vec{\boldsymbol{r}}_1 \times d\vec{\boldsymbol{r}}_2$$

by drawing a differential area element on an arbitrary surface and appealing to the geometric definition of the cross product as a directed area. Label the sides of the surface element with vectors $d\vec{\boldsymbol{r}}_1$ and $d\vec{\boldsymbol{r}}_2$ with both vectors' tails at the same point.

Similarly, derive the formula

$$d\tau = (d\vec{\boldsymbol{r}}_1 \times d\vec{\boldsymbol{r}}_2) \cdot d\vec{\boldsymbol{r}}_3$$

from a picture of an arbitrary differential volume element and the geometric definition of the scalar triple product as the volume of a parallelopiped. Label the sides of the volume element with vectors $d\vec{\boldsymbol{r}}_1$, $d\vec{\boldsymbol{r}}_2$, and $d\vec{\boldsymbol{r}}_3$ with all the vectors' tails at the same point. Make sure to choose a right-handed orientation.

Next, ask the students to use these formulas to find the surface and volume elements for a plane, for a finite cylinder (including the top and bottom), and for a sphere.

1.3 Student Conversations

- Students who do not have much experience with the "Use what you know" strategy have trouble getting from the generic expression for $d\vec{\boldsymbol{r}}$ in cylindrical and spherical coordinates to specific ones for the vectors that they care about. Encourage the students to draw pictures.

- A remarkable number of students have trouble finding the cross product. Emphasize that the curvilinear basis vectors ($\hat{\boldsymbol{s}}, \hat{\boldsymbol{\phi}}, \hat{\boldsymbol{z}}$) or ($\hat{\boldsymbol{r}}, \hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\phi}}$) are orthonormal, just like their rectangular conterparts: ($\hat{\boldsymbol{x}}, \hat{\boldsymbol{y}}, \hat{\boldsymbol{z}}$). You can do dot and cross products in curvilinear coordinates as long as the two vectors have their tails at the same point.

- A few students will want to jump to formulas that they know without actually doing the computation and/or they will want to simply multiply the lengths of the two $d\vec{\boldsymbol{r}}$s together. There is nothing wrong with this. You can save some time by going directly to these strategies. The only downside is that the students may not get any practice with the computational strategies that work in the (rarely needed) generic cases.