Student handout

Flux through a Cone

Find the flux through a cone of height H and radius R due to the vector field $\vec{F} = C\, z\, \hat{z}$.

1 Instructor's Guide

1.1 Introduction

This activity is part of a sequence on flux Flux Sequence, which we strongly recommend. If you don't have time, the minimum introduction is a short lecture introducing the concept of flux (as the amount of a vector field perpendicular to a surface) and how to calculate it:

$$\Phi = \int_S \vec{F} \cdot d\vec{A}$$

Prompt: Find the flux through a cone of height H and radius R due to the vector field $\vec{F} = C\, z\, \hat{z}$.

This prompt is open-ended in that it doesn't specify either the location of the cone or whether or not the circular top of the cone is to be considered part of the surface. We like to leave it open-ended, see what students do, and when students question the open-endedness, give a mini-"sermon" on the ill-posedness of most real world problems. If you are short of time, or otherwise want to avoid these questions, you should use a more explicit prompt.

If you choose the point of the cone at the origin (and allow it to open upward, like an icecream cone), then the problem can easily be solved in spherical coordinates as well as the obvious cylindrical coordinates.

1.2 Student Conversations

- **Choice of coordinates** - some groups will choose cylindrical and others will choose spherical coordinates and it can be done with either. Occasionally, a group will attempt to use Cartesian coordinates, but they usually realize quickly that this is not a good choice.
- **Into or Out of the Cone?** - the sign of the flux depends on whether we choose to calculate the flux up through the cone or down through the cone. Students need to be aware of this choice. The answers differ by a minus sign.
- **Evaluating the field on the surface** - in this case, the vector field varies along the surface of the cone so students must integrate over the surface rather than finding a constant flux times a surface area.
- **Finding the differential surface element** - students can find the differential surface element by taking the cross product of two $d\vec{r}$ vectors lying on the cone. Several issues arise, such as:
  - writing down the $d\vec{r}$'s using the "use what you know" strategy;

  - choosing the direction of the area element (i.e. the order of the vectors in the cross product);
  - making sure that the $d\vec{r}$ they choose actually lies on the cone.

- **Limits of integration** - some students have difficulty determining the limits of integration.

- **Only two parameters** - students will often forget to change all of the variables in the integrand into the two variables that are being integrated over.

1.3 Wrap-up

We do a brief summary of the main points to wrap up the activity.

- This is also good place to talk about the affordances of different choices for coordinates (e.g. ask a group that solved it in cylindrical and one that solved it in spherical to compare).

- It is important to reinforce the method of constructing the $d\vec{A}$ vector by taking the cross product $d\vec{r_1} \times d\vec{r_2}$.