**Student handout** Suppose you are standing on a hill. You have a topographic map, which uses rectangular coordinates $(x, y)$ measured in miles. Your global positioning system says your present location is at one of the following points (pick one):

$$A : (1, 4), \quad B : (4, -9), \quad C : (-4, 9), \quad D : (1, -4), \quad E : (2, 0), \quad F : (0, 3)$$

Your guidebook tells you that the height $h$ of the hill in feet above sea level is given by

$$h = a - bx^2 - cy^2$$

where $a = 5000$ ft, $b = 30\,\frac{\text{ft}}{\text{mi}^2}$, and $c = 10\,\frac{\text{ft}}{\text{mi}^2}$.

1. Starting at your present location, in what map direction (2-dimensional unit vector) do you need to go in order to climb the hill as steeply as possible? *Draw this vector on your topographic map.*
2. How steep is the hill if you start at your present location and go in this compass direction? *Draw a picture which shows the slope of the hill at your present location.*

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# 1 Instructor's Guide

## 1.1 Introduction

We preface this activity with a mini-lecture about the gradient. Students should be familiar with how to calculate a gradient:

$$\vec{\boldsymbol{\nabla}} f = \frac{\partial f}{\partial x}\,\hat{\boldsymbol{x}} + \frac{\partial f}{\partial y}\,\hat{\boldsymbol{y}} + \frac{\partial f}{\partial z}\,\hat{\boldsymbol{z}}$$

and the geometric property that the gradient points in the direction of greatest increase in the function.

## 1.2 Student Conversations

- Students very quickly figure out the location of the top of the hill and the height of the hill.
- **Where does the gradient live?** Students will not realize that, because the height is a function of two variables in this problem, the gradient of the height function is a 2-D vector that lives in the topo map.
- **Compass direction versus slope:** The gradient tells the students the direction of steepest ascent, and it also contains information about how quickly the height function is changing.

$$df = \vec{\nabla} f \cdot d\vec{r}$$

Therefore, the magnitude of the gradient is the slope. To find the 3-D vector direction of travel, students need to find a unit vector in the direction of gradient as well as the change in height. Most students will forget that they need a normalized vector in the x-y plane to give the 3-D vector pointing along the steepest direction at their point.

## 1.3 Wrap-up

The key understanding is that the gradient is always perpendicular to the level curves(for two dimensions) that they lie on. This is regardless of where the global maximum is of the function: if the level curves are not circular, then not everyone will be pointing towards the same point.

Have the students stand up and imagine the room is a hill, with the top of the hill in one of the corners of the room. Have the students close their eyes and use their rights arms to point in the direction of the gradient. Students should be alerted to the fact that their arms should be parallel to the floor. (See this activity.)