Number of Paths: For each of the fields, mark two different points A and B. Can you identify paths where the vector line integral has the indicated value (greater than, equal to, or less than zero)? In the table, enter the number of paths you can find for each condition (max 2 paths).

Field	Number of paths with $\int_A^B \vec{F}_i \cdot \vec{dr}$		
	> 0	$= 0$	< 0
$\vec{F}_1$			
$\vec{F}_2$			

Instructor's guide **Discussion: Breaking into pieces** Some students don't realize you can do piecewise paths (a curve connected to a straight line). Talk to these students right away.

Discussion: Choosing easier paths Discuss what paths are the most convenient for estimating the sign of the path integral and why.

Optional Further Discussion: A third field $\vec{F}_3$ could also be given to students. Its curl is zero everywhere except the center. It corresponds to a magnetic field for an infinite current carrying wire.

Match a Surface: One of the vector fields corresponds to (part of) your surface. Where does it match, and how do you know?

Instructor's guide **Student Reasoning:** The gradient is perpendicular to the level curves and points in the direction of increasing function. The line integral corresponds to a change in height on the surface.

Discussion: Gradient and Field Some students need to be told that the vector field is the gradient. The gradient will point in the direction of increase. Note that if the field is the electric field and the surface is the potential, there is a relative minus sign so that the field points in the direction of decreasing potential. This ideas is brought out much more strongly in the *Work and Electric Field* activity.

Extend to a New Surface: Could the other vector fields correspond to a surface? Explain why or

why not.

> **Instructor's guide** They could not, at least not in the typical way. This question can generate some interesting discussions about what information is encoded in scalar field.

Instructor's guide

SUMMARY PAGE

What Students Learn:

- Some differences between conservative and non-conservative fields
  - Line integrals on conservative fields are path independent.
  - Conservative fields are gradients of a scalar potential that can be represented as a surface; non-conservative fields cannot.

Time Estimate: 30 minutes

Equipment

- Red quadrupole surface
- Vector field handout in dry-erase sleeve for each group
- Dry-erase markers & erasers
- Whiteboard for each group
- Student handout for each student

Introduction

- Students should have some practice with vector line integrals. We suggest *Vector Integrals* as a warmup activity.

Whole Class Discussion / Wrap Up:

- Ask students to explain their reasoning and report their findings. Look for consensus/disagreement in the reasoning.