The internal energy of an ideal gas is described by the equation:

$$U(p,T) = \left(\frac{\#dof}{2}\right) NkT$$

The surfaces represent the internal energy of water vapor:

- The *blue* surface shows the internal energy as a function of temperature and pressure, $U(T,p)$, with volume and entropy contours etched into the surface.
- The *purple* surface shows the internal energy as a function of volume and entropy, $U(S,V)$, with pressure and temperatures contours etched into the surface.

By examining the plastic surfaces, how can you tell if an ideal gas model is a good model for water vapor?

Instructor's guide **Answer:** The biggest indicator on the p-T surface is if the surface is a plane. On the contour map, this corresponds to the internal energy contours being nearly parallel to the volume axis (constant at constant temperature) AND having the same, constant spacing between them. Also, volume contours are nearly linear (ideal gas law). For an ideal gas on the S-V graph, isotherms should be parallel to and never cross the U contours, i.e. on the purple surface the isotherms are level curves.

Discussion: Sketching a 2D graph of U vs T for an Ideal Gas When working with a contour map, students might benefit from sketching a graph of internal energy vs.temperature for an ideal gas, then sketching a plot of internal energy vs.temperature for water vapor, using the values from the contour map.

Discussion: Linearity The instructor might model the blue surface, describing the features of the surface, including its near-flatness. (Although it is not a perfectly flat, and this can be demonstrating by placing the surface upside down on a flat table, against a small whiteboard, etc.)

Discussion: Applications of the Model One can discuss degrees of freedom and gasses/situations that are typically modeled as ideal.

Instructor's guide If you give the students the scaling and the number of particles, they could be able to figure out the approximate degrees of freedom for water vapor (canonically, 9).

- 1kg water vapor is 55 mols of water
Internal Energy	2cm	$\rightarrow$	170. kJ
Temperature	2cm	$\rightarrow$	70 K

Instructor's guide

SUMMARY PAGE

Goals:

- Equation and definition of an Ideal Gas
- Students explore what it would mean for something to be an ideal gas or a good approximation of an ideal gas
- Students might discuss the difference between temperature and internal energy
- Can discuss degrees of freedom

Time Estimate: 15-30 minutes

Tools and Equipment:

- Purple and Blue Thermo surfaces for each group
- Remote Option: Ideal Gas Contour Maps
- Student handout for each student
- A personal or shared writing space for each student to write/draw/sketch

Intro:

- Students should be familiar with the basic definitions of pressure, volume, temperature, and internal energy

Whole Class Discussion:

- A whole class discussion should elicit student ideas about how to determine if a system fits the ideal gas model perfectly, and when the ideal gas model is a "good" fit.
- Compare and contrast how the ideal gas relationships appear on the blue vs purple surface. In particular, $U \propto T$ - blue surface is a plane; on the purple (and blue) surface the temperature and internal energy contours are parallel.