Orient Yourself to the Physical System & the Graph: The plot shows various thermodynamic quantities for water vapor in a piston (cylindrical thermos with a movable top) at different states. From state (point) A to state (point) B, estimate the following quantities:

Verbal Description	**Symbol**	**Estimate (with Units)**
Change in volume:	$\Delta V_{A\to B}$	
Change in entropy:	$\Delta S_{A\to B}$	
Change in temperature:	$\Delta T_{A\to B}$	
Change in pressure:	$\Delta P_{A\to B}$	
Change in internal energy:	$\Delta U_{A\to B}$	

Under what circumstances would you be willing to label these quantities with 'd's instead of Δ's? For example, dV instead of ΔV.

Instructor's guide **Discussion: What does "small" mean?** Some students say "small" but not what it is small compared to. The instructor can talk about linearity in the whole class discussion.

Discussion: Can you take a limit experimentally? There can be some good discussion about this (I'm not convinced there is a right answer, but is a good thing to think about.)

Discussion: Linearity Once the data is linear, the secant line becomes the tangent line and the slope is the derivative even if the measured changes are "large".

Determine a Rate: Pick two of the variables in the table and determine the rate of change of one with respect to the other from A to B. What experiment could you do to measure this rate?

Instructor's guide **Discussion: Holding Entropy Fixed** Between points A & B, the entropy doesn't change. The experiment the students describe should hold entropy constant, for example, by insulating the piston.

Discussion: Derivatives: "Under what circumstances would you be willing to call this rate a derivative?" Have students plot graphs of their functions to see how linear they are. Many students will not be willing to call this a derivative because we aren't taking a limit - talk about linear approximation.

Discussion: Derivative of which point? If a student is willing to call this rate a derivative, at which point is it the derivative? It is the best approximation to the derivative somewhere between the two points.

Discussion: Slope Is their derivative a slope? Of what?

> **Follow-up:** How would you represent this derivative symbolically? Some students write $\Delta U/\Delta T$ vs. dU/dT; some students write subscript S vs. $A \to B$. A good opportunity to talk about how the derivative ought to depend on the value of S.

Inverting Your Rate: Determine the reciprocal of the rate you calculated. Brainstorm a meaningful name for this rate.

> Instructor's guide **Discussion: Experiment** What experiment would you do to measure this quantity?
>
> **Discussion: Inversion** The derivative can be flipped as long as the path is unchanged.
>
> **Follow-up:** How would you represent this rate it symbolically?

Reversing the Path: How does the rate you previously calculated change if instead you went from state B to state A?

> Instructor's guide **Answer:** The rate (and therefore the derivative) is the same.
>
> **Discussion: Sign** Many students will initially say that the derivative changes sign.

Instructor's guide

SUMMARY PAGE

Goals

- To calculate derivatives on a contour map.
- Many different derivatives can be determined at a point.
- Think about derivatives as ratios of small changes.
- Reinforces the idea that a derivative is a ratio of small changes along a path
- In thermo, derivatives are often invertible?
- The sign of a derivative does not depend on the direction you're traveling along the path

Time Estimate: 30 minutes

Tools/Equipment

- Contour graph with P,V,S,T,U contours
- Student handout for each student
- A personal or shared writing space for each student to write/draw/sketch.

Intro

- Orient the students to the contour plot - it's busy.
- Exposure to differential calculus

Whole Class Discussion / Wrap Up

- How students report out their answers to Explore.
- Many students will not be willing to call the ratio of changes a derivative until you've taken the limit that the changes become infinitesimal. Talk about linear approximation and/or tables of data.
- Most students will have heard in a math class that derivatives are not invertible. In thermo, they often are as long as you don't change the path.
- Many students will think that if you reverse the path, the derivative will change sign. Discuss how the signs of both the numerator and denominator of the ratio of small changes swaps, leaving the derivative unchanged.