Start with a Simpler Case: The electrostatic potential due to a particle with charge q is:

$$V(r) = \frac{kq}{r}$$

where k is the electrostatic constant and r is the distance from the particle.

On your whiteboard, identify all the points with the same value of potential around a single point charge. Repeat for several different values of potential.

- What shapes have you drawn?
- If you wanted the difference in potential represented by the shapes to be equal, how are they spaced?

> Instructor's guide **Alternative:** This is nice as a small whiteboard question, where every student draws their answer on an individual whiteboard.
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> **Discussion:** This is a great place for a whole class discussion in the middle of an activity.
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> - **Inverse Function:** It is important that students understand that the distance between surfaces increases as you move away from the source (not equally spaced!) if you choose a convention that adjacent equipotential surfaces represent the same change in potential. It's helpful to draw a graph of an inverse function ($1/r$) and show how the graph tells you the spacing should get larger.
> - **Spherical Shells:** Point to a location in space off the board directly over the source - what is the value of the potential? What shape are the equipotential surfaces? (The surfaces are in fact nested spheres in space and not just the circles on the board.)
> - **Zero:** Where is the equipotential surface where $V = 0$? (Infinitely far away) What shape is this surface? (A spherical shell)

Add Complexity: Draw equipotential surfaces for the potential due to 4 particles with equal, positive charge arranged in a square.

> Instructor's guide **Prep:** For the next part, it is helpful to use a template to draw the position of the particles so that the scale matches the dry-erasable surface. We recommend also doing the same here (even though there is no surface).
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> **Student Discussions:**
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> - **Forces vs. Potentials:** Students tend to want to make arguments about forces rather than potentials. They will therefore tend to conclude incorrectly that the potential at the center of

the square is zero. Remind students that force is related to how the potential **changes**, and that the derivative of a function can be zero at a location where the value of the function is not zero.

- **Far Away from the Source:** Students usually quickly realize that, far away, the square looks like a point charge and the surfaces are spheres.
- **Distance and Fall Off:** Students will need to think about how the potential falls off faster nearer the charge and is essentially constant far away from the charge.
- **Field Lines:** Some students will confuse electric field lines and equipotential curves. It helps to discuss the single point charge case. Affirm that equipotential surfaces are perpendicular to the field lines and encourage superposition/fall-off kinds of reasoning.

Potential Pitfall: We have seen students try to draw 3D perspective drawings of a graph of V. Everyone has trouble interpreting these drawings, including the drawer. It is worth talking to students about what they are trying to represent and then show them the Mathematica notebook.

Discussion: This is also a good place to interrupt the activity - students tend to think the spacing is closest in the middle of square and are surprised by the correct spacing (the spacing is closest on the side away from the center—there is a local minimum at the center of the square). Help the students figure this out by thinking about fall-off.

Mathematica Notebook: After this section, demonstrating the Mathematica notebook for this activity is useful, as is discussing the many different ways to represent a 3D scalar field.

Examine a New Case: Repeat for a quadrupole: 2 positively charged particles and 2 negatively charged particles arranged in a square, with "like" charged particles on opposite corners.

Instructor's guide **Student Discussions:**

- **Surfaces for $V = 0$:** If students are floundering, it is helpful to direct their attention to places where the potential is zero. It bothers some students that the equipotential sphere at infinity intersects the V=0 symmetry planes. Some students remember from their intro course or from a math course that equipotential surfaces/level curves should not intersect.
- **Surface Model:** Some students have trouble seeing the V=0 lines on the surface. using a small credit/business card to visualize the straight, flat lines helps.
- **Sign of the Charge:** This example is more difficult than the previous one because now students have to take into consideration the sign of the potential when adding them together.
- **Using Multiple Representations:** We have observed students using the Mathematica notebook and the surface to "look up the answer" and then try to explain it. We encourage this kind of behavior as a way for the students to get themselves unstuck.

Extend to New Surfaces: The red surface represents the potential of a quadrupole in the plane of the charges (at $z = 0$ cm). What would the potential look like in the $z = 1$ cm plane? What would be different? What about the $z = -1$ cm?

> **Instructor's guide** The set of surfaces for the quadrupole includes surfaces for "$z = 1$" and "$z = 2$". The Mathematica notebook also has visualizations for the potential outside the plane of the charges.

Instructor's guide

SUMMARY PAGE

What Students Learn:

- Add the potential due to each charge to calculate the potential due to a collection of charges.
- Equipotential surfaces are 3D surfaces where the potential is a constant value.
- The spacing between equipotential surfaces, by convention, is such that the change in potential is the same for adjacent equipotential surfaces.
- Therefore, close spacing means the potential is changing quickly with distance; wide spacing means the potential is changing slowly.
- Considering equipotential surfaces is only one of many ways to visualize the electric potential in space.
- Inverse square force law means that the potential changes faster closer to the source—far away, the potential changes slowly.

Time estimate: 100 - 120 minutes

Equipment

- Red quadrupole surface
- Mathematica Notebook
- Dry-erase markers & erasers
- Tabletop whiteboard for each group
- Paper template for marking locations of charged particles
- Student handout for each student
- Orange and Yellow quadrupole stack

Whole Class Discussion / Wrap Up:

- The hardest part of this activity is figuring out the spacing in the middle of the distribution. Have a few groups share how they thought about this. Beware of electric field/force types of reasoning - students tend to think incorrectly that balanced forces mean the potential is zero. Try to encourage superposition/fall-off types of reasoning over force types of reasoning.
- Beware - we're using the word "surfaces" in two contexts here. The surface manipulatives are not equipotential surfaces.

- Use the quadrupole stack to help students visualize the equipotential surfaces in 3D. First ask students what they think these surface represent - some students can guess! The blue and green dots represent the same value of potential as each other and on each of the three surfaces. The shells are egg shaped - they can also be visualized with the Mathematica notebook.