Orient Yourself to the Physical System & the Graph: The surface represents measurements of internal energy on a kilogram of water vapor in a piston (a graduated cylinder with a moveable top). The purple surface graph is $U(S, V)$.

Make Predictions: Starting at the red star, as you increase the volume of the system, what happens to the internal energy of the water vapor?

> Instructor's guide **Answer:** The internal energy could increase, decrease, or stay the same. It depends on how the volume is increasing (and what is happening to the entropy).
>
> **Student Ideas:** Most students will interpret "increasing volume" as "hold the entropy fixed". This is, of course, a possibility, but it is not the only possible way to increase the volume. From their math classes, many students will have the idea that for a partial derivative, everthing else must be held constant.
>
> **Discussion: Alternative Graph** Could you use the other surface? If so, how?
>
> **Discussion: Does it matter where you start?** Does your answer depend on the initial state of the system
>
> **Helping Students Understand the Graphs:** Students may need guidance to recognize the meaning of the contour lines, and the fact that they can use either surface to answer this question (although one is much easier that the other).

Consider a Special Case: Is it possible to change the volume without changing the internal energy? (Is your answer consistent with your answer to the previous prompt?)

> Instructor's guide **Answer:** Yes, it is possible - you move along a level curve of the internal energy. Along this level curve, both volume and entropy change.
>
> **Discussion: Checking Consistency** Some students will happily answer "internal energy increases" to the first prompt and "yes it is possible to change the volume and not change the internal energy" to the second prompt without resolving the inconsistency between the two answers.

Signs of Partial Derivatives: Using the purple surface graph, determine if the following derivatives are positive, negative, or zero.

$$\left(\frac{\partial U}{\partial V}\right)_S \qquad \left(\frac{\partial U}{\partial V}\right)_T \qquad \left(\frac{\partial U}{\partial V}\right)_p$$

Instructor's guide

SUMMARY PAGE

Goals

- For functions of two independent variables, you need to specify changes in both independent variables to determine how the function changes.
- Can lead into a lecture about partial derivatives and specifying a path.

Time Estimate: 15 minutes

Tools/Equipment

- A Purple $U(S, V)$ Plastic Surface for each group
- Dry-erase markers

Intro

- Students need to be oriented to the surfaces and what they represent.

Whole Class Discussion:

- The main take-away is that for functions of two variables, you need to specify a change in both the independent variables to know the change in the function.
- You can ask student groups to share an example of a path where the volume increases and the internal energy: increases, decreases, and is constant.
- This activity leads into ideas about (1) how the number of independent variable you have in a thermal system corresponds to the number of ways of getting energy into or out of a system, (2) you need to specify the path for a partial derivative, and (3) for thermal system, you actually CANNOT "hold all the other variables constant".