Estimate Work on A Path: The attached vector field represents an electric field. Draw a path starting at the red star and ending at the purple triangle. Estimate the work done on a test charge by the field along the path you drew. The longest vector shown on the vector field has a magnitude of 15 electric field units.

Instructor's guide **Prep:** Change the beginning and end points to vary the level of difficulty of the integration as needed.

Answer:

1. Chop, Multiply, & Add (what we most want): Select a path tangent to the electric field, chop up the path, estimate the distance between points and magnitudes of electric fields, multiply, and add.

2. Estimation with Average Value: Estimate an average electric field along the path, estimate the length of the path, and multiply.

3. **Potential Pitfall:** Students may incorrectly find the change in potential by saying that the lengths of the arrows are the values of the potential.

Potential Pitfall: Students may not consider units. This is a good place to discuss how distance on the vector map can mean physical distance or electric field magnitude.

Discussion: Units We have intentionally left off SI units for electric field. For electric field, students can either use N/C, which helps students to think about the electric field as force per charge, or V/m, which helps students to think about the electric field as the gradient of the potential. Discussing both is useful.

Path Independence: Some students have trouble distinguishing the path independence of the work done by a conservative field and the path independence of the total displacement. These students will argue that the work done on two different paths is the same because the total displacement is the same. It is nice to have a non-conservative field on hand to give to these students to demonstrate that, although the displacement is path independent, the work is not always path independent.

Relate Representations: How is the work done by the electric field related to the surface?

Instructor's guide **Discussion: Gradient and Slope** This is a good place to talk about how the electric field is the gradient(slope) of the surface. You're multiplying the slope by a "run" to get a "rise".

Discussion: Most students know that electric field vectors point toward negative charges and that the negative charge lies at the bottom of a potential well. However, most students are really surprised by the negative sign in the relationship: $\int \vec{E} \cdot \vec{dr} = -\Delta V$. This is the big bang for this activity.

Missing /var/www/paradigms_media_2/media/conservative_field_plotAA_plot-20-z0mm-field.png

Instructor's guide

SUMMARY PAGE

What Students Learn:

- Students practice doing a vector line integral of an electric field.
- Students match the electric field to a surface representing the potential and see that a positive line integral of the electric field corresponds to a drop in the potential (a negative change) $\int \vec{E} \cdot \vec{dr} = -\Delta V$

Time Estimate: 20 min

Introduction

- Students should have practice with doing vector line integrals. We suggest the *Vector Integrals* activity as a warmup.
- Students should know that the electric field in the negative gradient of the electric potential.

Whole Class Discussion / Wrap Up:

- Emphasize the *Chop, Multiply, Add* approach to doing the vector line integral. Allow several groups to share how they performed the integral.
- Talk about how since the electric field is the gradient (slope) of the potential, the line integral corresponds to multiplying the slope by the "run" to get the "rise".
- But there is an overall negative sign, so a positive slope corresponds to a "drop".