A *pressure cooker* is an enclosed pot that expels air and traps water vapor, which increases the internal pressure. This in turn raises the boiling point of water and allows food to cook at high temperatures.

Imagine you have a large industrial pressure cooker that holds 1 kg of water vapor. You would like to know how responsive the system is to changes in temperature. To do this, you need to determine a characteristic *rate*: the ratio of the amount of heat needed to change the temperature a small amount and the change in temperature (i.e., the amount of heat transferred per change in temperature).

The graph provided by your instructor shows internal energy and volume contours plotted on temperature and pressure axes.

1. **Estimate the Temperature-Responsiveness:** Use the graph to determine this temperature-responsiveness when the volume is held fixed. The initial state of the system corresponds to the black square. Describe your process.

   Instructor's guide **Student Ideas:** Students might need help in recognizing that for a pressure cooker, you can control the pressure and temperature but the volume of water vapor is constant.

   Answer: We want the ratio of change in internal energy and the change in temperature along the path of constant volume. It is the ratio of small changes, not a slope on this graph.

   Discussion: Which derivative? We haven't yet told students what derivative to estimate. Have a whole class discussion about this after most groups have struggled for a few minutes and have had some insights. It's nice to have done the Quantifying Change activity before this one. Some correct answers might look like: $đQ/dT$, $(dU/dT)_V$, $Q/\Delta T$, $(\Delta U/\Delta T)_V$.

   Student Ideas: Students try to use equipartition theorem or other equations, rather than directly measuring the rate on the graph. Students might not always realize that work done is zero, or how this would relate U and Q.

   Follow-Up: For what graph would the slope be the rate you're looking for? How is that graph related to the surface?

2. **Reflect on Your Process:** Why do you think it matters that you held volume constant in the above estimate?

   Instructor's guide **Answer:** Two reasons: (1) you need to specify a path in order to take the derivative, and (2) holding the volume constant means the work done is zero, so the heat added is equal to the change in internal energy.

3. **Explore Dependencies on State Variables:** Does the value of your estimate depend on the value of the volume? the temperature?

   Instructor's guide Any answer is productive here. For an ideal gas, the answer is no. Water vapor is nearly an ideal gas (but not quite). The heat capacity at constant volume is not the directional derivative.

   Instructor's guide **Optional Extensions:**

   - How would you estimate the heat capacity at constant PRESSURE?

     Answer: The most straightforward way is to account for the work done along the constant pressure path and subtract that from the change in internal energy.)

     Which value is bigger: C_V or C_p? Why do you think that is?

   - What procedure would you use to you estimate C_v from the purple surface?

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SUMMARY PAGE

Goals

- Heat capacity at constant volume relates to changes in internal energy, i.e. $đQ = dU$.
- A conceptual definition of heat capacity at constant volume is the derivative of internal energy with respect to temperature without changing the volume.
- Not all derivatives are slopes but they are all ratios of small changes.
- Heat capacity at constant volume depends on the value of the volume.
- **Optional:** Heat capacity at constant pressure is NOT $(dU/dT)_p$ – you have to account for work done.

Equipment Needed:

- Heat Capacity contour map
- Student worksheet for each student
- A personal or shared writing space for each student to write/draw/sketch.
- **Optional:** Blue $U(T, p)$ plastic surface for each group

Introduction

- Students should have seen 1st Law of Thermodynamics and the Thermodynamic Identity
- Students should know how to compute work as $p\, dV$

Whole Class Discussion:

- Students need to figure out what derivative they're trying to estimate. Having an early WCD about this would be useful.
- The derivative they are trying to estimate is tantalizingly close to a slope, especially for the blue dot and the green triangle. Not all partial derivatives are slopes but they are all ratios of small changes.
- For an ideal gas, the heat capacities are constant. Water vapor is not an ideal gas but it is pretty close. In general, the heat capacity is a state variable.