In polar coordinates, the position vector is:

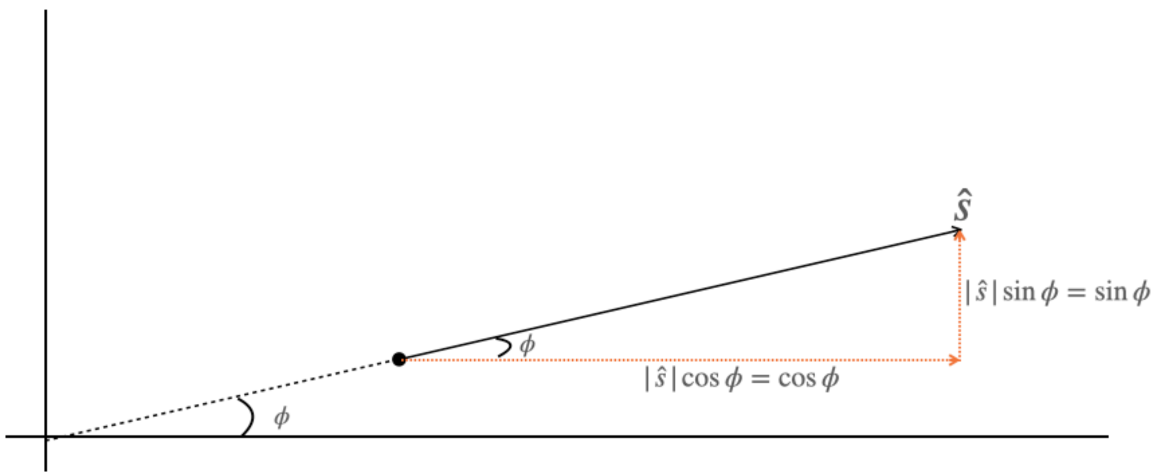
$$\vec{r} = s\hat{s}$$

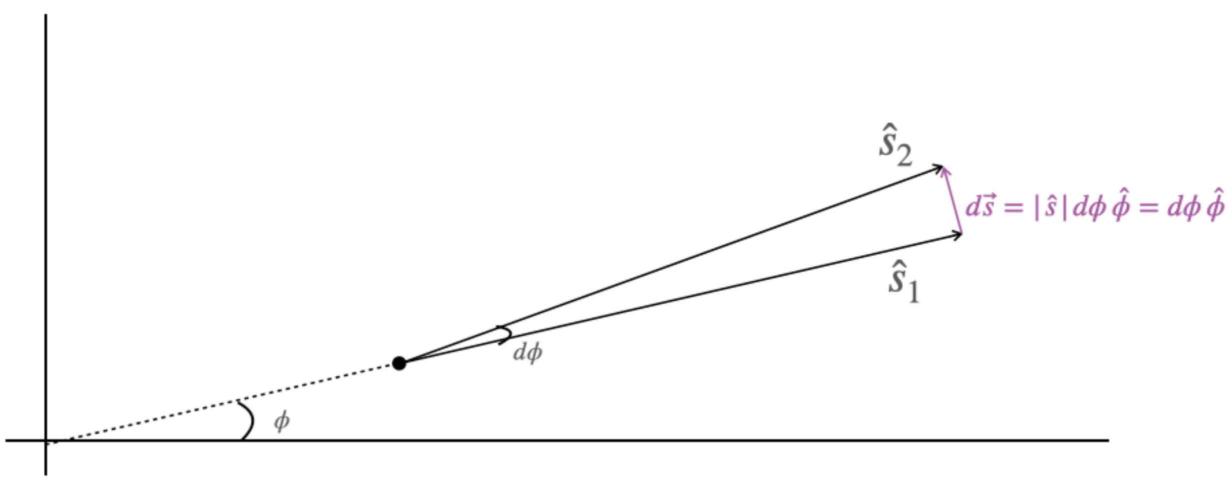
To find the velocity, take a time derivative:

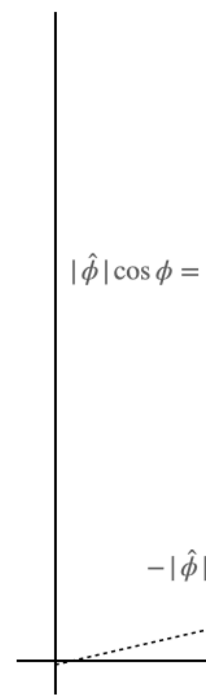
$$\begin{aligned}\vec{v} &= \frac{d}{dt}(s\hat{s}) \\ &= \dot{s}\hat{s} + s\dot{\hat{s}}\end{aligned}$$

But what is $\dot{\hat{s}}$?
Start by writing $\hat{s}$ and $\hat{\phi}$ using Cartesian basis vectors:

$$\begin{aligned}\hat{s} &= \cos\phi\hat{x} + \sin\phi\hat{y} \\ \hat{\phi} &= -\sin\phi\hat{x} + \cos\phi\hat{y}\end{aligned}$$







Now, take the time derivative:

$$\frac{d}{dt}\hat{s} = -\sin\phi\dot{\phi}\hat{x} + \cos\phi\overset{0}{\cancel{\dot{\hat{x}}}} + \cos\phi\dot{\phi}\hat{y} + \sin\phi\overset{0}{\cancel{\dot{\hat{y}}}}$$
$$= \dot{\phi}\hat{\phi}$$

$$\frac{d}{dt}\hat{\phi} = -\cos\phi\dot{\phi}\hat{x} - \sin\phi\overset{0}{\cancel{\dot{\hat{x}}}} - \sin\phi\dot{\phi}\hat{y} + \cos\phi\overset{0}{\cancel{\dot{\hat{y}}}}$$
$$= -\dot{\phi}\hat{s}$$

So, plugging back into the velocity:

$$\vec{v} = \dot{s}\hat{s} + s\dot{\hat{s}}$$
$$\vec{v} = \dot{s}\hat{s} + s\dot{\phi}\hat{\phi}$$

Now, I can find the acceleration the same way:

$$\vec{a} = \frac{d}{dt}\left(\dot{s}\hat{s} + s\dot{\phi}\hat{\phi}\right)$$
$$= \ddot{s}\hat{s} + \dot{s}\dot{\hat{s}} + \dot{s}\dot{\phi}\hat{\phi} + s\ddot{\phi}\hat{\phi} + s\dot{\phi}\dot{\hat{\phi}}$$
$$= \ddot{s}\hat{s} + \dot{s}(\dot{\phi}\hat{\phi}) + \dot{s}\dot{\phi}\hat{\phi} + s\ddot{\phi}\hat{\phi} + s\dot{\phi}(-\dot{\phi}\hat{s})$$
$$= (\ddot{s} - s\dot{\phi}^2)\hat{s} + (s\ddot{\phi} + 2\dot{s}\dot{\phi})\hat{\phi}$$

How can I interpret these acceleration terms?

$$\vec{a} = (\overbrace{\ddot{s}}^{\text{acceleration of radial coordinate}} \underbrace{-s\dot{\phi}^2}_{\text{centripetal acceleration}})\hat{s} + (\overbrace{s\ddot{\phi}}^{\text{tangential acceleration}} + \underbrace{2\dot{s}\dot{\phi}}_{\text{Coriolis acceleration}}$$

- The first term tells me the **second derivative of the radial coordinate**.
- The second term is pointing toward the center (it has a minus sign) and is proportional to the square of the angular velocity. This is like $a_c = r\omega^2 = \frac{v^2}{r}$. This is the **centripetal acceleration**. When something moves in a circle, in order to keep turning (and move in a straight line with constant speed), there needs to be an acceleration towards the center.
- The third term is the **tangential acceleration**, like $a_T = r\alpha$.

- The fourth term is a **Coriolis acceleration**. This is an acceleration that arises from the radial coordinate changing while the angular coordinates changes (e.g., the object moves away from the center while also moving around the center).