Student handout

Time Evolution of the Infinite Square Well

Launch the "Quantum Bound States" PhET. Pause the simulation at t=0.
Make the square well potential as deep as you can on the screen to approximate an infinite well.

1. What happens to the energy levels if you:

   a) change the width of the well?
   b) change the mass of the particle?

   How your observations are consistent with the equation for the energy eigenvalues?

2. At t=0, what do the energy eigenstate wavefunctions look like?

   a) The real part?
   b) The imaginary part?
   c) The magnitude?
   d) The phase?

   How do these shapes and colors make sense?

3. At t=0, what does the probability density look like for the energy eigenstates?

   a) How is the shape of the probability density related to the shape of the wavefunction?
   b) For the n=2 energy eigenstate (in other words, the first excited state)
      i. If you were looking for the particle in the box, where is the particle most likely to be? Explain.
      ii. What is the expectation value of the position of the particle? Explain.

4. As time passes, what do the energy eigenstates do? (What do they look like?)

   a) The real part?
   b) The imaginary part?
   c) The magnitude?
   d) The phase?

   Explain why you see what you see.

5. As time passes, what does the probability density of the energy eigenstates look like?

   Explain why you see what you see.

6. Create a superposition state:

   a) What does the wavefunction look like at t=0?

   b) How does the wavefunction evolved with time?

   c) How does the probability density evolved with time?

   Explain why you see what you see.

1 Introduction

This activity is meant to give students meaningful grounding into the different pieces of wavefunctions and how they fit together visually. This also shows in a highly customizable way the varience in wave functions that can be created through superpositions of eigenstates and how the energy interplays with what the wavefunction is doing and how it is progressing in time.

2 Student Conservations

1. **Wow!:**Some groups make a game of finding cool superposition states that evolve with time in cool patters. Try to stear them towards making explanations rather than just having a "wow, cool" reaction.

2. **Tunneling:** Some students will notice that the wavefunction bleeds a little on the edges, particularly is the well is really wide. They are noticing tunneling, which is a nice sneak peak that they will come back to in Ph427 (Periodic Systems).

3. **Stationary and Non-Stationary States:** Once into the superposition steps, there can be a lot of variables, encourage them to experiment with different combinations and evaluate what things are similar and disimilar from stationary states. What can they count on now and what can they not in general?

3 Wrap Up

Hightlighting how the energy levels of the well change as we change m (mass) and L (size of the box) could be handled as a quick wrap up near the beginning of the activity but might be unecessary. A mid-activity wrap up discussing the $t = 0$ parts is advised so students are all on the same page going into time evolution, where the key details should be:

1. How the probability density relates to the wavefunction's real and imaginary parts (e.g., the probability density includes information from both.)

2. How the phase is indicated in the program (e.g. from the real with red being pure real and light blue being about $\frac{\pi}{2}$ off real or pure imaginary).

3. Clarifying that the probability density is the thing we use to determine probabilities for meaurements of position, not the real or imaginary wavefunctions in isolation.

4. Why stationary states are stationary (i.e. all the time dependence in the coefficents comes out in the wash when you take the norm sqaure of ψ)

The final wrap up at the end of the activity shoould focus on the variablility present in the superposition of wave functions and how the change in probability density indicates we are no longer in a stationary state and we should expect our probabilities of position meaurements in particular regions of the well to change.