

Student handout

Gauss's Law:

$$\oint \vec{E} \cdot \hat{n} dA = \frac{1}{\epsilon_0} Q_{\text{enc}}$$

Ampère's Law:

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 I_{\text{enc}}$$

Potentials:

$$\begin{aligned}\vec{E} &= -\vec{\nabla} V \\ \vec{B} &= \vec{\nabla} \times \vec{A}\end{aligned}$$

Maxwell's Equations:

$$\begin{aligned}\vec{\nabla} \cdot \vec{E} &= \frac{\rho}{\epsilon_0} \\ \vec{\nabla} \cdot \vec{B} &= 0 \\ \vec{\nabla} \times \vec{E} &= 0 \\ \vec{\nabla} \times \vec{B} &= \mu_0 \vec{J}\end{aligned}$$

Superposition Laws:

$$\begin{aligned}V(\vec{r}) &= \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}') d\tau'}{|\vec{r} - \vec{r}'|} \\ \vec{E}(\vec{r}) &= \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}')(\vec{r} - \vec{r}') d\tau'}{|\vec{r} - \vec{r}'|^3} \\ \vec{A}(\vec{r}) &= \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}') d\tau'}{|\vec{r} - \vec{r}'|} \\ \vec{B}(\vec{r}) &= \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}') \times (\vec{r} - \vec{r}') d\tau'}{|\vec{r} - \vec{r}'|^3} \\ V(B) - V(A) &= - \int_A^B \vec{E} \cdot d\vec{r}\end{aligned}$$

Position Vectors

$$\begin{aligned}
\vec{r} &= x\hat{x} + y\hat{y} + z\hat{z} \\
&= s\hat{s} + z\hat{z} \\
&= r\hat{r}
\end{aligned}$$

The distance between two position vectors

1. In cylindrical coordinates:

$$|\vec{r} - \vec{r}'| = \sqrt{s^2 + s'^2 - 2s s' \cos(\phi - \phi') + (z - z')^2}$$

2. In spherical coordinates:

$$|\vec{r} - \vec{r}'| = \sqrt{r^2 + r'^2 - 2r r' [\sin \theta \sin \theta' \cos(\phi - \phi') + \cos \theta \cos \theta']}$$

Derivatives in Rectangular Coordinates:

$$\begin{aligned}
\vec{\nabla} f &= \frac{\partial f}{\partial x} \hat{x} + \frac{\partial f}{\partial y} \hat{y} + \frac{\partial f}{\partial z} \hat{z} \\
\vec{\nabla} \cdot \vec{F} &= \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \\
\vec{\nabla} \times \vec{F} &= \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \right) \hat{x} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x} \right) \hat{y} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right) \hat{z}
\end{aligned}$$

Derivatives in Cylindrical Coordinates:

$$\begin{aligned}
\vec{\nabla} f &= \frac{\partial f}{\partial s} \hat{s} + \frac{1}{s} \frac{\partial f}{\partial \phi} \hat{\phi} + \frac{\partial f}{\partial z} \hat{z} \\
\vec{\nabla} \cdot \vec{F} &= \frac{1}{s} \frac{\partial}{\partial s} (s F_s) + \frac{1}{s} \frac{\partial F_\phi}{\partial \phi} + \frac{\partial F_z}{\partial z} \\
\vec{\nabla} \times \vec{F} &= \left(\frac{1}{s} \frac{\partial F_z}{\partial \phi} - \frac{\partial F_\phi}{\partial z} \right) \hat{s} + \left(\frac{\partial F_s}{\partial z} - \frac{\partial F_z}{\partial s} \right) \hat{\phi} + \frac{1}{s} \left(\frac{\partial}{\partial s} (s F_\phi) - \frac{\partial F_s}{\partial \phi} \right) \hat{z}
\end{aligned}$$

Derivatives in Spherical Coordinates:

$$\begin{aligned}
\vec{\nabla} f &= \frac{\partial f}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial f}{\partial \theta} \hat{\theta} + \frac{1}{r \sin \theta} \frac{\partial f}{\partial \phi} \hat{\phi} \\
\vec{\nabla} \cdot \vec{F} &= \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 F_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta F_\theta) + \frac{1}{r \sin \theta} \frac{\partial F_\phi}{\partial \phi} \\
\vec{\nabla} \times \vec{F} &= \frac{1}{r \sin \theta} \left(\frac{\partial}{\partial \theta} (\sin \theta F_\phi) - \frac{\partial F_\theta}{\partial \phi} \right) \hat{r} + \frac{1}{r} \left(\frac{1}{\sin \theta} \frac{\partial F_r}{\partial \phi} - \frac{\partial}{\partial r} (r F_\phi) \right) \hat{\theta} \\
&\quad + \frac{1}{r} \left(\frac{\partial}{\partial r} (r F_\theta) - \frac{\partial F_r}{\partial \theta} \right) \hat{\phi}
\end{aligned}$$

Lorentz Force Law:

$$\vec{F} = q_{\text{test}} \left(\vec{E} + \vec{v} \times \vec{B} \right)$$

Step and Delta Functions:

$$\begin{aligned}
\frac{d}{dx} \theta(x-a) &= \delta(x-a) \\
\int_{-\infty}^{\infty} f(x) \delta(x-a) dx &= f(a)
\end{aligned}$$

Vector Calculus Theorems:

$$\begin{aligned}
\oint \vec{F} \cdot d\vec{A} &= \int \vec{\nabla} \cdot \vec{F} d\tau \\
\oint \vec{F} \cdot d\vec{\ell} &= \int (\vec{\nabla} \times \vec{F}) \cdot d\vec{A}
\end{aligned}$$

Total Charge and Current:

$$\begin{aligned}
Q &= \int \rho(\vec{r}') d\tau' \\
I &= \int \vec{J}(\vec{r}') \cdot d\vec{A}'
\end{aligned}$$