Student handout

Electrostatic Potential from Two Charges

- Find a formula for the electrostatic potential $V(\vec{r})$ that is valid everywhere in space for:
  - Two charges $+Q$ and $+Q$ placed on the z-axis at $z' = D$ and $z' = -D$.
  - Two charges $+Q$ and $-Q$ placed on the z-axis at $z' = D$ and $z' = -D$, respectively.
- Simplify your formulas for the special cases of:
  - the x-axis
  - the z-axis
- Discuss the relationship between the symmetries of the physical situations and the symmetries of the functions in these special cases.

1 Instructor's Guide

1.1 Introduction

It may help to do the `https://paradigms.oregonstate.edu/act/2053` activity before this one. There is an alternative version of this activity `https://paradigms.oregonstate.edu/act/2076` in which students find series expansions of the potential along the axes of symmetry.

Students typically know the iconic formula for the electrostatic potential of a point charge $V = \frac{kq}{r}$. We begin this activity with a short lecture/discussion that generalizes this formula in a coordinate independent way to the situation where the source is moved away from the origin to the point $\vec{r}'$, $V(\vec{r}) = \frac{kq}{|\vec{r}-\vec{r}'|}$. (A nice warm-up (SWBQ) to lead off the discussion:

Introductory SWBQ Prompt: "Write down the electrostatic potential everywhere in space due to a point charge that is not at the origin." The lecture should also review the superposition principle.

$$V(\vec{r}) = k\sum_i \frac{q_i}{|\vec{r} - \vec{r}_i|}$$

This general, coordinate-independent formula should be left on the board for students to consult as they do this activity.

1.2 Student Conversations

- Students create an expression such as

$$V(x, y, z) = kQ\left(\frac{1}{|z - D|} + \frac{1}{|z + D|}\right)$$

Each axis and charge distribution has a slightly different formula. A few groups may have trouble coordinatizing $|\vec{r} - \vec{r}'|$ into an expression in rectangular coordinates, but because the coordinate system is set up for them, most students are successful with this part fairly quickly.

- Many students are likely to treat this as a two-dimensional case from the start, ignoring the y axis entirely. Look for expressions like

$$V = k \sum_{i=1}^{N} \frac{q_i}{\sqrt{x^2 + (z - z_i)^2}}.$$

  Prompt them to consider the larger, 3-dimensional picture.

- Most students will leave off the absolute value signs when evaluating the potential on the z-axis. If they do this, their formulas will not be correct for negative values of z. The subtlety here is that

$$\sqrt{a^2} = |a|$$

  not

$$\sqrt{a^2} = a$$

  in contexts like this when $\sqrt{a^2}$ is a distance and therefore necessarily positive and when a itself might be either positive or negative.