Student handout The following are 2 different representations for the **same** state on a quantum ring

$$|\Phi\rangle = \sqrt{\frac{1}{2}}\,|2\rangle - \sqrt{\frac{1}{4}}\,|0\rangle + i\sqrt{\frac{1}{4}}\,|{-2}\rangle \tag{1}$$

$$\Phi(\phi) \doteq \sqrt{\frac{1}{8\pi r_0}}\left(\sqrt{2}e^{i2\phi} - 1 + ie^{-i2\phi}\right) \tag{2}$$

1. Write down the matrix representation for the same state.
2. With all 3 representations, calculate the probability that a measurement of L_z will yield $0\hbar$, $-2\hbar$, $2\hbar$.
3. If you measured the z-component of angular momentum to be $2\hbar$, write down the full resultant state immediately after the measurement.
4. If an energy measurement is performed on the state $\Phi(\phi)$, what is the probability that the energy measurement will yield each of the following values: $0\frac{\hbar^2}{I}$?,$2\frac{\hbar^2}{I}$?,$4\frac{\hbar^2}{I}$?
5. If you measured the energy of the state to be $2\frac{\hbar^2}{I}$, write down the full resultant state immediately after the measurement.

1 Instructor's Guide

1.1 Introduction

This activity is long, but covers a lot of important concepts for this course all together. Now that students have seen the 3 different types of representations in PH425, we want them to see the parallels of calculating the probabilities in each representation so they can begin to see how they each say the same thing about the state.

1.2 Student Conversations

- This is the first activity where we ask them to write down a column vector represenation for a state on a new system. This will often take quite a while, while student wrestle with where each piece of information from the state goes in that representation. This struggle is worth letting play out, but if students get stuck, writing a column vector with inner products or c_m's inside it can help considerably.
- Letting students struggle with how to index the rows of the column vector is also productive. Allowing them to do it the opposite way of McIntyre can be a good learning experience because fundamentally how to index the column vector is a personal choice.

- It will have probably been some time since students have done a projection postulate or "state immediately after a measurement" problem. Many students will still be assumingthe state will always callapse into 1 eigenstate. Others will know it becomes a superposition, but will drop the phase information and/or assume the probabilities of measuring each state will default to $\frac{1}{2}$, our probabilties have been constructed to catch this error, be sure to look out for it.

- This is the first calculation they'll be doing in the sequence of the course which involves degeneracy, which is a huge concept going foraward. Some students will do the right thing automatically, but many more will be confused, this is a place where explaining is more pertinent than letting them struggle. They will have several times to get this right in front of the teaching team, but getting as many students on the same page early is paramount since the degeneracy gets more and more complicated as we go to the sphere and hydrogen.

- Students still struggling with the mathematics will frequently write statements where they sum the coefficents INSIDE the norm sqaure, looking something like this $\mathcal{P}_{2\frac{\hbar^2}{I}} = |c_2 + c_{-2}|^2$ instead of $\mathcal{P}_{2\frac{\hbar^2}{I}} = |c_2|^2 + |c_{-2}|^2$, which can be easy to overlook if you're not watching for it. Students will sometimes still get the right answer, implying that they think they can distribute the norm sqaure onto individual terms, this will run them into trouble later on.

- Little things like what probabilities look like mathematically (norm sqaure of the inner product) and subtlies like not forgetting to take the complex conjugate of a bra in wavefunction and matrix notation is something students will still having issues with. Be vigilant about asking groups questions when you're seeing them pause for periods of time during calculations and reinforce sensemaking for measurements and probabilities since they have some distance from QM content and are in the process of recall here.

1.3 Wrap-up

It is strongly advisable to interuppt the students as the class progresses through the questions. Again, this is a long activity with a lot of important take aways.

1. The first wrap up should occur after most of the class has a solid column vector, write out the correct column vector, explaining how and why you're indexing the way you are. Be sure to mention how other choices are valid, but m's starting positive at the top of the vector and going to negative at the bottom is the McIntyre convention and the one that will be used in this course. Some students will also not make the connection that it is the probability amplitudes, the c's or the inner products of the state on each eigenstate, that make up the rows of the column vector, be sure to point this out explicitly, they will be asked to do this again for the Sphere and Hydrogen, and also for operators where indexing gets more complicated.

2. The second interruption is optional, but if students are having difficulty remembering how to do probabilities for L_Z, it is worth stopping them and showing it explicity before they handle degenerate cases.

3. The third interuption should be a final tying together of the degenerate cases for probabilties and states after measurements. Explcitly writing out the sum of probabilties and giving a process for

finding the state formally is a most (don't have to use the full projection postulate, but you should namecheck it, since students should recognize it).

1.4 Extensions

This activity overlaps with activity 5 quite a bit, since the probabilities end up being time independent in part 1 of this activity, where they are not in part 2 (part 2). If this activity ends up being too long, you can move the coverage of degeneracy to after you cover time dependence in the next couple days. As many examples of students practicing calculations in class, especially with degeneracy, is better though.