Recall that $U = \frac{3}{2}Nk_BT$ for a monatomic gas.

$$\Omega = CV^N U^{\frac{3}{2}N} \tag{1}$$

$$\vdots \tag{2}$$

$$U = \frac{3}{2}Nk_BT \tag{3}$$

The equipartition theorem is an elegant shortcut, but takes some steps to practice.

Step 1 Choose a set of parameters that can describe any arbitrary state of the material (gas or otherwise). This will include positions and velocities, and could also include angular momenta. If the molecule contains "springs" you might want to include the lengths of those springs.

Step 2 Write an explicit expression for the total classical Newtonian energy of the material in terms of the independent free variables.

For the ideal monatomic gas, we have

$$E = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \frac{1}{2}mv_3^2 + \cdots \text{(total of } N \text{ atoms)}$$

where if we write things in terms of Cartesian coordinates gives us

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_{1x}^2 + \frac{1}{2}mv_{1y}^2 + \frac{1}{2}mv_{1z}^2$$

so you end up with $3N$ terms that depend on independent free variables.

Step 3 Count the number of indepdendent free variables that are squared in the expression for the energy ("quadratic terms"). We call this the **number of degrees of freedom** f.

For the ideal monatomic gas $f = 3N$.

Step 4 The equipartition theorem predicts that

$$U_{\text{classical}}(T) = \frac{f}{2}k_BT \tag{4}$$

For the ideal monatomic gas $U_{\text{classical}}(T) = \frac{3}{2}Nk_BT$.

For the curious, here is a 3.5 minute video explaining ineptly why only the quadratic terms in the energy get equipartition, recorded in Spring 2021.

> **Instructor's guide** After this we do something else.
> Here is a video of me explaining ineptly why only the quadratic terms in the energy get equipartition.