Instructor's guide Follows Quantum mechanical correction to the equipartition theorem.

In the prompt, explain that 50, 500 and 5000 should be equally spaced, for instance. 2, 5, and 10 are also approximately equally spaced.

As it turns out, the heat capacity is about $\frac{5}{2}$ for almost the entire temperature range.

For N_2 gas molecules in a 10 cm cubic box, the rules of QM dictate the discrete allowed values for:

1. Translational K.E. in one dimension: $\{1 \times 10^{-40}\ \mathrm{J}, 4 \times 10^{-40}\ \mathrm{J}, 9 \times 10^{-40}\ \mathrm{J}, \ldots\}$
2. Rotational K.E.: $\{0\ \mathrm{J}, 0.8 \times 10^{-22}\ \mathrm{J}, 0.8 \times 10^{-22}\ \mathrm{J}, 0.8 \times 10^{-22}\ \mathrm{J}, 2.5 \times 10^{-22}\ \mathrm{J}, \ldots\}$

Instructor's guide The bond length of N_2 is 1.1 Å. Its mass is $m = 14$ amu $\approx 2 \times 10^{-26}$ kg. The moment of inertia is thus

$$I = 2 \cdot 2 \times 10^{-26}\ \mathrm{kg} \left(0.55 \times 10^{-10}\ \mathrm{m}\right)^2 \tag{1}$$
$$\approx 1.2 \times 10^{-46}\ \mathrm{kg} \cdot \mathrm{m}^2 \tag{2}$$

From which we can find the energy eigenvalues:

$$E_\ell = \frac{\hbar^2}{2I}\ell(\ell+1) \tag{3}$$
$$= \frac{(10^{-34}\ \mathrm{J} \cdot \mathrm{s})^2}{2 \cdot 1.2 \times 10^{-46}\ \mathrm{kg} \cdot \mathrm{m}^2}\ell(\ell+1) \tag{4}$$
$$\approx \ell(\ell+1) \cdot 0.4 \times 10^{-22}\ \mathrm{J} \tag{5}$$

3. Vibrational energy: $\{2.3 \times 10^{-20}\ \mathrm{J}, 6.9 \times 10^{-20}\ \mathrm{J}, 11.5 \times 10^{-20}\ \mathrm{J}, \ldots\}$

Instructor's guide I looked up the experimental vibrational frequency, which is 2358 cm^{-1}.

Sketch a graph of $\frac{dU}{dT}$ of 10^{22} molecules of N_2 gas in the temperature range of 70 K (the temperature at which N_2 becomes liquid at 1 atm of pressure) to 5000 K (at which temperature N_2 breaks apart).

(use a logarithmic temperature axis)