Student handout

1. In the small town of Coriander, the library can be found by starting at the center of the town square, walking 25 meters north ($\vec{\boldsymbol{a}}$), turning 90° to the right, and walking a further 60 meters ($\vec{\boldsymbol{b}}$).
   - Draw a figure showing the displacement vectors $\vec{\boldsymbol{a}}$ and $\vec{\boldsymbol{b}}$, as well as their sum, the displacement vector $\vec{\boldsymbol{v}} = \vec{\boldsymbol{a}} + \vec{\boldsymbol{b}}$.
   - How far is the library from the center of the town square?
   - Let $\hat{\boldsymbol{x}}$ be the unit vector pointing east, and $\hat{\boldsymbol{y}}$ be the unit vector pointing north. Express $\vec{\boldsymbol{a}}$, $\vec{\boldsymbol{b}}$, and $\vec{\boldsymbol{v}}$ in terms of $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{y}}$.
2. It turns out that magnetic north in Coriander is approximately 14° degrees east of true north. If you use a compass to find the library (!), the above directions will fail. Instead, you must walk 39 meters in the direction of magnetic north ($\vec{\boldsymbol{A}}$), turn 90° to the right, and walk a further 52 meters ($\vec{\boldsymbol{B}}$).
   - Draw a figure showing the displacement vectors $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$, as well as their sum, the displacement vector $\vec{\boldsymbol{v}} = \vec{\boldsymbol{A}} + \vec{\boldsymbol{B}}$.
   - How far is the library from the center of the town square?
   - Let $\hat{\boldsymbol{X}}$ be the unit vector pointing towards "magnetic east", and $\hat{\boldsymbol{Y}}$ be the unit vector pointing towards magnetic north. Express $\vec{\boldsymbol{A}}$, $\vec{\boldsymbol{B}}$, and $\vec{\boldsymbol{v}}$ in terms of $\hat{\boldsymbol{X}}$ and $\hat{\boldsymbol{Y}}$.
3. Can any vector displacement within the town limits be expressed as the sum of two vectors, one of which points north and the other east?
4. **FOOD FOR THOUGHT:** Where on Earth is Coriander?!

0.1 Essentials

0.1.1 Main ideas

- The same physical vector can be written in terms of more than one basis.

0.1.2 Prerequisites

- The geometric definition of vectors as arrows in space.
- The geometric definition of vector addition.

0.1.3 Warmup

- None — this is a nice activity for the first day of class.

- You might need to start off with a discussion of small group activities, and the roles you expect students to play.

0.1.4 Props

- whiteboards and pens
- aviation maps if available
- globe

0.1.5 Wrapup

- Need notation that refers explicitly to the basis.
- Discuss reasons for change in notation from $\langle a, b\rangle$ to $a\,\hat{\boldsymbol{x}} + b\,\hat{\boldsymbol{y}}$
- $+$ is really *vector* addition.

Many students will equate vectors with their components. Emphasize that vectors are geometric objects; they do not depend on a choice of coordinates or basis vectors. Writing "$\langle a, b\rangle$" hides these essential choices by assuming the existence of a preferred (rectangular) coordinate system. Not only does this usage conceal the existence of this underlying choice, it also makes it difficult to compare different choices. For this reason, we *always* write out vector components as coefficients of explicit basis vectors, such as $a\,\hat{\boldsymbol{x}} + b\,\hat{\boldsymbol{y}}$. (See also the section on Subsidiary Ideas below.)

0.2 Details

0.2.1 In the Classroom

This activity is straightforward; students have few problems with it.

Question 3 asks whether any vector displacement be expressed as the sum of a vector pointing east and one pointing north. Many students are bothered by this, since they don't know whether they can use negative coefficients. We have deliberately left the question ambiguous in order to generate a short discussion about conventions. To the lay person, South is a different direction from North; in mathematics, these are not independent directions. We nonetheless accept both answers, provided a reasonable explanation is given.

Get students to answer the first two questions on the board — on the same diagram. Watch out! The second student often uses a different scale than the first; the resulting vectors won't agree. Not all students realize this is a problem! Asking how many libraries there are usually resolves this issue.

A good question to ask at the end is which representation of the vector displacement to the library is the most important. (The answer of course is that it depends on the question.) Most students use the Pythagorean Theorem to get the length in #1. Fancy trigonometry, even rotation matrices, appear for #2. All of these approaches are fine, but makes discussing various methods afterward all the more important. The dot product should be mentioned. The location of Coriander is interesting. Asking students the magnetic declination of their hometown can get the ball rolling.

0.2.2 Subsidiary Ideas

- **Multiple bases**

  When working with more than one basis, it is essential to incorporate the basis explicitly into the notation. In particular, using the common notation $\langle x, y, z\rangle$ for the components of a vector should be discouraged, not only because there is no indication of which basis one is using, but also because it makes it difficult for some students to realize the difference between vectors and scalars. (This issue is not helped by the fact that the components of vectors are *not* scalars!)

- All 2-dimensional vectors can be written in terms of 2 basis vectors.

- Components can be negative.

- The definitions of orthogonal, normalized, and orthonormal.

- **Geometric definition of dot product**

  This would be a good place to introduce or reinforce the geometry of the dot product, and especially the fact that $|\vec{\boldsymbol{v}}|^2 = \vec{\boldsymbol{v}} \cdot \vec{\boldsymbol{v}}$;

0.2.3 Enrichment

- Where is Coriander?
  In other words, what locations on Earth have the correct angle between geographic and magnetic north?

- NOAA has a website which can be used to determine the magnetic deviation for any location: `https://paradigms.oregonstate.eduhttps://www.ngdc.noaa.gov/geomag/calculators/magcalc.shtml`

- It is an interesting problem in spherical geometry to find the set of points for which the magnetic deviation is constant. A globe helps! *To the best of our knowledge the solution to this problem is not known.*