Student handout For each of the following vector fields, find a potential function if one exists, or argue that none exists.

- $\vec{\boldsymbol{F}} = (3x^2 + \tan y)\,\hat{\boldsymbol{x}} + (3y^2 + x\sec^2 y)\,\hat{\boldsymbol{y}}$
- $\vec{\boldsymbol{G}} = y\,\hat{\boldsymbol{x}} - x\,\hat{\boldsymbol{y}}$
- $\vec{\boldsymbol{H}} = (2xy + y^2\sin z)\,\hat{\boldsymbol{x}} + (x^2 + z + 2xy\sin z)\,\hat{\boldsymbol{y}} + (y + z + xy^2\cos z)\,\hat{\boldsymbol{z}}$
- $\vec{\boldsymbol{K}} = yz\,\hat{\boldsymbol{x}} + xz\,\hat{\boldsymbol{y}}$

0.0.1 Main ideas

- Finding potential functions.

Students love this activity. Some groups will finish in 10 minutes or less; few will require as much as 30 minutes. [1]

0.0.2 Prerequisites

- Fundamental Theorem for line integrals
- The Murder Mystery Method

0.0.3 Warmup

none

0.0.4 Props

- whiteboards and pens

0.0.5 Wrapup

- Revisit integrating conservative vector fields along various paths, including reversing the orientation and integrating around closed paths.

[1] More accurately, students love the Murder Mystery Method! We often incorporate this activity into an exam review, rather than devoting an entire period to it.

0.1 Details

0.1.1 In the Classroom

- We recommend having the students work in groups of 2 on this activity, and not having them turn anything in.
- Most students will treat the last example as 2-dimensional, giving the answer xyz. Ask these students to check their work by taking the gradient; most will include a $\hat{\boldsymbol{z}}$ term. Let them think this through. The correct answer of course depends on whether one assumes that z is constant; we have deliberately left this ambiguous.
- It is good and proper that students want to add together multivariable terms. Keep returning to the gradient, something they know well. It is better to discover the guidelines themselves.

0.1.2 Subsidiary ideas

- 3-d vector fields do not necessarily have a $\hat{\boldsymbol{z}}$-component!

0.1.3 Homework

A challenging question to ponder is why a surface fails to exist for nonconservative fields. Using an example such as $y\,\hat{\boldsymbol{x}}+\hat{\boldsymbol{y}}$, prompt students to plot the field and examine its magnitude at various locations. Suggest piecing together level sets. There is serious geometry lurking that entails smoothness. Wrestling with this is healthy.

0.1.4 Essay questions

Write 3-5 sentences describing the connection between derivatives and integrals in the single-variable case. In other words, what is the one-dimensional version of MMM? Emphasize that much of vector calculus is generalizing concepts from single-variable theory.

0.1.5 Enrichment

- The derivative check for conservative vector fields can be described using the same type of diagrams as used in the Murder Mystery Method; this is just moving down the diagram (via differentiation) from the row containing the components of the vector field, rather than moving up (via integration). We believe this should not be mentioned until after this lab.

  When done in 3-d, this makes a nice introduction to curl — which however is not needed until one is ready to do Stokes' Theorem. We would therefore recommend delaying this entire discussion, including the 2-d case, until then.
- Work out the Murder Mystery Method using polar basis vectors, by reversing the process of taking the gradient in this basis.
- Revisit the example in the Ampère's Law lab, using the Fundamental Theorem to explain the results. This can be done without reference to a basis, but it is worth computing $\vec{\boldsymbol{\nabla}}\phi$ in a polar basis.