Student handout Consider the region D in the xy-plane shown below, which is bounded by

$$u = 9 \qquad u = 36 \qquad v = 1 \qquad v = 4$$

where

$$u = xy \qquad v = \frac{y}{x}$$

If you want to determine x and y as functions of u and v, consider uv and u/v.

Missing `/var/www/paradigms_media_2/media/activity_media/covBnew.png`

- List as many methods as you can think of for finding the area of the given region.
  It is enough to refer to the methods by name or describe them briefly.
- For at least 3 of these methods, give explicitly the formulas you would use to find the area.
  You must put limits on your integrals, but you do not need to evaluate them.
- Using any 2 of these methods, find the area.
  One of these should be a method we have learned recently.
- Now consider the following integral over the same region D:
  $\iint_D \frac{y}{x}\ dA$
- Which of the above methods can you use to do this integral?
- Do the integral.

0.0.1 Main ideas

- There are many ways to solve this problem!
- Using Jacobians (and inverse Jacobians)

0.0.2 Prerequisites

- Surface integrals
- Jacobians
- Green's/Stokes' Theorem

0.0.3 Warmup

Perhaps a discussion of single and double integral techniques for solving this problem.

0.0.4 Props

- whiteboards and pens

0.0.5 Wrapup

This is a good conclusion to the course, as it reviews many integration techniques. We emphasize that (2-dimensional) change-of-variable problems are a special case of surface integrals.

Here are some of the methods one could use to do these integrals:

- change of variables (at least 2 ways)
- Area Corollary to Green's Theorem (at least 2 ways)
- ordinary single integral (at least 2 ways)
- ordinary double integral (at least 2 ways)
- surface integral

0.1 Details

0.1.1 In the Classroom

- Some students will want to simply use Jacobian formulas; encourage such students to try to solve this problem both by computing $\frac{\partial(x,y)}{\partial(u,v)}$ and by computing $\frac{\partial(u,v)}{\partial(x,y)}$.
- Other students will want to work directly with $d\vec{\boldsymbol{r}}_1$ and $d\vec{\boldsymbol{r}}_2$. This works fine if one first solves for x and y in terms of u and v.
- Students who compute $d\vec{\boldsymbol{r}}_1$ and $d\vec{\boldsymbol{r}}_2$ directly can easily get confused, since they may try to eliminate x or y, rather than u or v.

  Along the curve $v =$ constant, one has $dy = v\,dx$, so that $d\vec{\boldsymbol{r}}_1 = dx\,\hat{\boldsymbol{x}} + dy\,\hat{\boldsymbol{y}} = (\hat{\boldsymbol{x}} + v\,\hat{\boldsymbol{y}})\,dx$, which some students will want to write in terms of x alone. But one needs to express this in terms of du! This can be done using $du = x\,dy + y\,dx = x(v\,dx) + y\,dx = 2y\,dx$, so that $d\vec{\boldsymbol{r}}_1 = (\hat{\boldsymbol{x}} + v\,\hat{\boldsymbol{y}})\,\frac{du}{2y}$. A similar argument leads to $d\vec{\boldsymbol{r}}_2 = (-\frac{1}{v}\,\hat{\boldsymbol{x}} + \hat{\boldsymbol{y}})\,\frac{x\,dv}{2}$ for $u =$ constant, so that $d\vec{\boldsymbol{S}} = d\vec{\boldsymbol{r}}_1 \times d\vec{\boldsymbol{r}}_2 = \hat{\boldsymbol{z}}\,\frac{x}{2y}\,du\,dv = \hat{\boldsymbol{z}}\,\frac{du\,dv}{2v}$. This calculation can be done without solving for x and y, provided one recognizes v in the penultimate expression.

  Emphasize that one must choose parameters, both on the region, and on each curve, and that u and v are chosen to make the limits easy.
- Take time before the activity to gauge students' recollection of single variable techniques and the Jacobian. After the activity, be sure to set up more than one approach. People will be fine after the first couple of steps but shouldn't leave class feeling stuck.

0.1.2 Subsidiary ideas

- Review of Green's Theorem
- Review of single integral techniques
- Review of double integral techniques

0.1.3 Enrichment

- Discuss the 3-dimensional case, perhaps relating it to volume integrals.