Instructor's guide These lecture notes are week 8 of `https://paradigms.oregonstate.edu/courses/ph441`.

Note that the figures come from `https://paradigms.oregonstate.edu/media/figures/sankey-engines.py`

1 Week 8:Work, heat, and cycles (K&K 8, Schroeder 4)

This week we will be zooming through chapters 8 of Kittel and Kroemer. Chapter 8 covers heat and work, which you learned about during *Energy and Entropy*. Hopefully this will be a bit of review and catch-up time, before we move on to phase transitions.

1.1 Heat and work

As we reviewd in week 1, heat and work for a quasistatic process are given by

$$Q = \int T dS \tag{1}$$

$$W = -\int p dV \tag{2}$$

But we can often make use of the First Law in order to avoid computing both of these (if we know how to find the internal energy):

$$\Delta U = Q + W \tag{3}$$

1.1.1 Carnot cycle

We have a monatomic ideal gas, and you can use any of its properties that we have worked out in class. We can begin with what you saw in Energy and Entropy

$$pV = NkT \tag{4}$$

$$U = \frac{3}{2}NkT \tag{5}$$

and we can add to that the results from this class:

$$S = Nk\left(\ln\left(\frac{n_Q}{n}\right) + \frac{5}{2}\right) \tag{6}$$

$$F = NkT\left(\ln\left(\frac{n}{n_Q}\right) - 1\right) \tag{7}$$

$$n = n_Q e^{-\beta\mu} \tag{8}$$

$$n_Q \equiv \left(\frac{mkT}{2\pi\hbar^2}\right)^{\frac{3}{2}} \tag{9}$$

$$ \tag{10}$$

Let us consider a simple cycle in which we start with the gas at temperature T_C.

1. Adiabatically compress the gas until it reaches temperature T_H.
2. Expand a gas to twice its original volume at fixed *temperature* T_H.
3. Expand the gas at fixed *entropy* until its temperature reaches T_C.
4. Finally go back to the original volume at *fixed temperature* T_C.

Small groups Solve for the heat and work on each of these steps. In addition find the total work done.

Answer We can solve this problem most easily by working out the heat at each step.

1. Since the process is adiabatic, $Q_1 = 0$. To find the work, we just need to know $\Delta U = \frac{3}{2}Nk\Delta T$. So the work must be $W = \frac{3}{2}Nk\Delta(T_H - T_C)$.
2. Now we are increasing the volume, which will change the entropy. Since the temperature is fixed, $Q = T\Delta S$, and we can find ΔS easily enough from the Sackur-Tetrode entropy: $\Delta S = Nk \ln 2$. Since the internal energy doesn't change, the heat and work are opposite. $Q = -W = NkT_H \ln 2$.
3. Now we are again not changing the entropy, and thus not heating the system, so $W = \Delta U$, and the work done is equal and opposite of the work done on step #1. $W = \frac{3}{2}Nk\Delta(T_C - T_H)$.
4. This will be like step 2, but now the temperature is different, and since we are compressing the work is positive while the heat is negative: $Q = -W = NkT_C \ln \frac{1}{2} = -NkT_C \ln 2$.

Putting these all together, the total work done is

$$W = NkT_H \ln 2 - NkT_C \ln 2 \tag{11}$$
$$= \ln 2Nk(T_H - T_C) \tag{12}$$

1.1.2 Efficiency of an engine

If we are interested in this as a heat engine, we have to ask what we put into it. This diagram shows where energy and entropy go. The engine itself (our ideal gas in this case) returns to its original state after one cycle, so it doesn't have any changes. However, we have a hot place (where the temperature is T_H, which has lost energy due to heating our engine as it expanded in step 2), and a cool place at T_C, which got heated up when we compressed our gas at step 4. In addition, over the entire cycle some work was done.

Missing /var/www/paradigms_media_2/media/activity_media/carnot-engine.pdf

Figure 1: Carnot engine energy and entropy flow diagram. The entropy lost by the hot place is the same as the entropy gained by the cold place, because the Carnot engine is reversible.

The energy we put in is all the energy needed to keep the hot side hot, which is the Q for step 2.

$$Q_H = NkT_H \ln 2 \tag{13}$$

The efficiency is the ratio of what we get out to what we put in, which gives us

$$\varepsilon = \frac{W}{Q_H} \tag{14}$$
$$= \frac{\ln 2 N k (T_H - T_C)}{N k T_H \ln 2} \tag{15}$$
$$= 1 - \frac{T_C}{T_H} \tag{16}$$

' and this is just the famous Carnot efficiency.

Note I could have made this an easier problem if I had changed the statement to expand at fixed temperature until the entropy changed by a given ΔS. Then we would not have had to use the Sackur-Tetrode equation at all, and our result would have been true for *any* material, not just an ideal gas!

We could also have run this whole cycle in reverse. That would look like the next figure. This is how a refridgerator works. If you had an ideal refridgerator and an ideal engine with equal capacity, you could operate them both between the inside and outside of a room to acheive nothing. The engine could precisely power the refridgerator such that no net heat is exchanged between the room and its environment.

Missing `/var/www/paradigms_media_2/media/activity_media/carnot-fridge.pdf`

Figure 2: Carnot fridge energy and entropy flow diagram.

Naturally, we cannot create an ideal Carnot engine or an ideal Carnot refridgerator, because in practice a truly reversible engine would never move. However, it is also very useful to know these fundamental limits, which can guide real heat engines (e.g. coal or nuclear power plants, some solar power plands) and refridgerators or air conditioners. Another use of this ideal picture is that of a heat pump, which is a refridgerator in which you cool the outside in order to heat your house (or anything else). A heat pump can thus be more efficient than an ordinary heater. Just looking at the diagram for a Carnot fridge, you can see that the heat in the *hot* location exceeds the work done, preciesly because it also cools down the cold place.