Introduction A good introduction to this activity is a SWBQ asking "What is Integration".

Student Conversations Some students will try to integrate the volume, writing something like $\int_0^H \pi r^2 H\, dH$. This would be a good time to introduce them to dimensional analysis; this expression incorrectly goes like L^4. Most students will try to chop the cylinder into circles. Many will then try to integrate the area of the circle, obtaining an integral of the form $\int_0^H \pi r^2\, dr$. Asking these students to think about what is changing and what is not may help them realize that the integral should be $\int_0^H \pi r^2\, dh$, as also will evaluating these two integrals and realizing that only the latter gives the correct answer. Some students may turn the cylinder on its side, so that the height corresponds to x; this is essentially the formalism used in first-year calculus classes for surfaces of revolution. Some students may try to chop the cylinder into vertical rectangles. Most such groups will have difficulty figuring out how to add up the resulting pieces, although a few students may remember the formula $\frac{1}{2}r^2\phi$ for the area of a piece of pie with angle ϕ and radius r. (This formula can be derived by dividing the area of a circle into pie-shaped pieces.) A few students might try to chop the cylinder into smaller cylinders. It is difficult to add up such pieces, as that would require coordinating the thickness of the sides of these pieces with the thickness of the top and bottom.

Wrap-up Ask each group to present one of their integrals, ideally one that differs from those used by previous groups. The instructor may wish to keep track of the different integrals by drawing a picture of the appropriate chopping in each case.

Things to emphasize during the wrap-up:

Integration is about chopping, multiplying, and adding. The first step is therefore to decide how to chop. The second step is to determine the volume of each piece. The volume of a small piece is small! Adding up the pieces is usually the easy part. For example, chopping the cylinder into "circles" really means chopping the cylinder into thin "disks" (really cylinders), whose volume is the area of the cirle (πr^2) times the height (dh). Adding up these pieces now requires determining the limits of integration and evaluating an easy integral.

Extensions This activity leads naturally into a discussion of finding the total amount of chocolate on a straight pretzel (e.g. Pocky) by chopping the pretzel into pieces of length dx, then adding up the amounts $\lambda\, dx$ of chocolate on each piece.

Student handout A cylinder has circular base of radius R and height H, both measured in feet.

1. What is the volume of the cylinder?
2. Write down as many different integrals as you can for computing this volume.
3. Do at least two of these integrals.

For some integrals, you may wish to use the fact that

$$\cos(2\alpha) = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha$$