Student handout

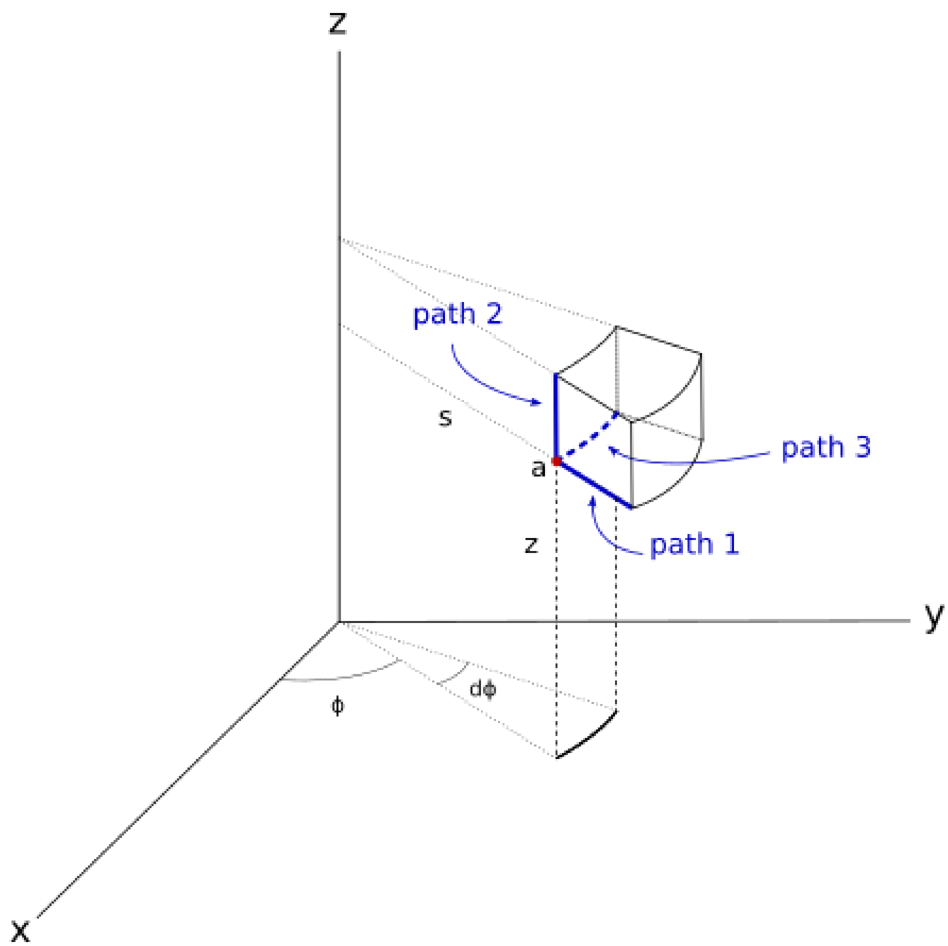
Cylindrical Coordinates Using the first figure below, determine the **length** $d\ell$ of each of the three paths shown (the three thick lines). Notice that, along any of these three paths, only one coordinate s, ϕ, or z is changing at a time (i.e. along path 1, $dz \neq 0$, but $d\phi = 0$ and $ds = 0$).

Path 1: $d\ell =$

Path 2: $d\ell =$

Path 3: $d\ell =$

Use your results to determine the volume of the region.
$d\tau =$



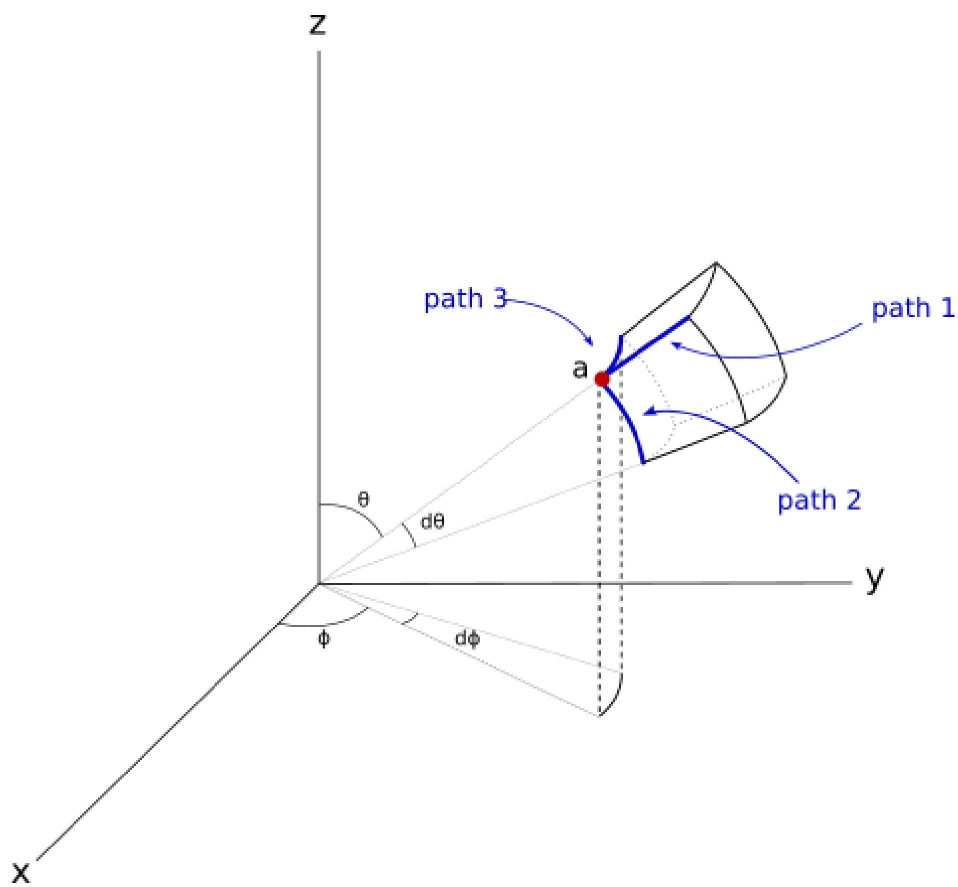
Spherical Coordinates Using the second figure below, determine the **length** $d\ell$ of each of the three paths shown (the three thick lines). Notice that, along any of these three paths, only one coordinate r, θ, or ϕ is changing at a time (i.e. along path 1, $d\theta \neq 0$, but $dr = 0$ and $d\phi = 0$).
(Be careful: One path is trickier than the others.)

Path 1: $d\ell =$

Path 2: $d\ell =$

Path 3: $d\ell =$

Use your results to determine the volume of the region.
$d\tau =$



1 Instructor's Guide

1.1 Main Ideas

This activity allows students to discover formulas for $d\ell$ in cylindrical, and spherical coordinates, using purely geometric reasoning.

1.2 Students' Task

Using a picture as a guide, students write down an algebraic expression for infinitesimal lengths in two different coordinate systems (cylindrical, spherical).

1.3 Introduction

Begin by drawing a curve (like a particle trajectory, but avoid "time" in the language) and an origin on the board. Show the position vector $\vec{r}$ that points from the origin to a point on the curve and the position vector $\vec{r} + d\vec{r}$ to a nearby point. Show the vector $d\vec{r}$ and explain that it is tangent to the curve.

1.4 Student Conversations

For the case of cylindrical coordinates, students who are pattern-matching will write $d\ell = d\phi$ on path 3. Point out that ϕ is dimensionless and that path three is an arc with arclength $s\,d\phi$.

Some students will remember the formula for arclength, but many will not. The following sequence of prompts can be helpful.

- What is the circumference of a circle?
- What is the arclength for a half circle?
- What is the arclength for the angle $\frac{\pi}{2}$?
- What is the arclength for the angle ϕ?
- What is the arclength for the angle $d\phi$?

For the spherical case, students who are pattern matching will now write $d\ell = r\,d\phi$. It helps to draw a picture in cross-section so that they can see that the circle whose arclength gives the coefficient of $d\phi$ has radius $r \sin\theta$. It can also help to carry around a basketball to write on to talk about the three dimensional geometry of this problem.

1.5 Wrap-up

The only wrap-up needed is to make sure that all students have (and understand the geometry of!) the correct formulas for $d\ell$.

1.6 Extensions

This is one of several similar activities using infinitesimal reasoning in curvilinear coordinates. Unlike most of the others, this one does not use $d\boldsymbol{\vec{r}}$, but only its magnitude $d\ell = |d\vec{r}|$. The $d\vec{r}$ version of this activity is `https://paradigms.oregonstate.edu/act/2071`.