Student handout

Systems of Equations: Compare and Contrast

Small Group Directions

1. Solve your assigned system of equations using any algebraic method. Show you work and be ready to explain how you solved it.
2. Also graph the system of equations and show how the solution appears on your graph. You may use graphing technology such as Desmos.

Group Roles

Facilitator: Read the directions out loud and check whether everyone understands each other. *"How should we start?" "How do you know?"*
Team Captain: Help your team members step up and step back. *"How do you know?" "What do you think?"*
Resource Manager: Help your group get unstuck. *"Is this working?" "What else could we try?" "Should we ask a team question?"*
Recorder/Reporter: Be prepared to share out in the whole class discussion. *"How should I explain...?"*

Problems

1. $$y = -3x4x + y = 2$$
2. $$y = 7x - 52x + y = 13$$
3. $$x = -5y - 4x - 4y = 23$$
4. $$x + y = 10y = x - 4$$
5. $$y = 5 - x4x + 2y = 10$$
6. $$3x + 5y = 23y = x + 3$$
7. $$y = -x - 22x + 3y = -9$$
8. $$y = 2x - 3 - 2x + y = 1$$
9. $$x = \frac{1}{2}y + \frac{1}{2}2x + y = -1$$

10.
$$a = 2b + 4b - 2a = 16$$

11.
$$y = 3 - 2x4x + 2y = 6$$

12.
$$y = x + 1x - y = 1$$

(Adapted from CPM *Core Connections*)

Whole Class Directions

1. Each group will share out how you solved your system of equations.
2. Listen to each group and think about similarities and differences.
3. Ask questions about anything you do not understand or you disagree with.
4. You do not need to write anything during the whole class discussion, but you will have an exit ticket to see what you learned from the discussion.

Exit Ticket: Systems of Equations Compare and Contrast

Sheila missed class today. She tried to solve Problem 8 on her own, but she thinks she made a mistake because -3 does not equal 1.

$$y = 2x - 3 \quad (1)$$
$$-2x + y = 1 \quad (2)$$

$$-2x + (2x - 3) = 1 \quad (3)$$
$$-2x + 2x - 3 = 1 \quad (4)$$
$$0 - 3 = 1 \quad (5)$$
$$-3 = 1 \quad (6)$$

Explain to Sheila what happened, using as much detail as possible to help her understand this type of problem.

Introduction

This Compare and Contrast activity is based on the College Preparatory Mathematics (CPM) *Core Connections Algebra* Parent Guide with Extra Practice, freely available here. CPM is a problem based curriculum with many conceptual problems for students to work on in small groups in class. The parent guide provides examples, exercises, and solutions for students to work alone and/or with parent support if they miss class or need extra practice. As such, the parent guide is one aspect of the CPM curriculum *most* focused on practice of procedures. The attached problem set is copied exactly from the CPM Parent Guide; the surrounding student instructions were written by Alyssa Sayavedra.

Special Cases of note

Problems 8 and 12 have no solution while Problem 11 has infinite solutions. It is important to include these problems, but be prepared for small groups to get tripped up by them. Many students, when solving equations, expect the "answer" to be a value. They may struggle to interpret an equation that is always or never true.

All other problems have one solution with integer coordinates.

Some problems in this set are easier than others. If any group finishes early, they can be encouraged to complete a second problem. Problem 1 is the most straightforward since y is equal to only one term. The next easiest problems are 2, 3, 4, and 8, because they do not require distribution after substitution.

Problems 1, 2, 3, 4, 5, 8, 9 and 11 can be solved using the Equal Values Method without introducing new fractions. The Equal Values Method is a variant of substitution in which students solve both equations for the same variable, then set the equations equal to each other, resulting in a single equation in one variable. This method is easier for many students because it results in a simpler one variable equation and is less prone to distribution errors. But it is usually not worth introducing fractions into the problem in order to use this method.

Problems 5, 9 and 11 can be simplified by either multiplying or dividing an entire equation by 2. It is unusual for students to think of this strategy at this stage, but it can be a helpful preview of the elimination method. This method also removes the fractions in Problem 9.

Small variations in notation can easily trip students up. Problem 10 uses a and b instead of x and y. Problems 3 and 9 have one equation solved for x instead of y. Problems 4 and 6 have the second equation solved for y instead of the first. Do not be surprised if some students still solve the first equation for y and plug it into the second.

Suggestions for Facilitating Small Group Work

Remind students of class norms for productive and respectful group work. Assign one problem to each group, including at least problems 2, 4, 6, 8, 11 and 12. Walk once or twice around the class within the first five minutes to make sure all small groups understand how to get started and are making progress. Make sure students understand the directions and have started to dig into the mathematics, but avoid giving strategic suggestions at this stage. The purpose of the small group time is for students to wrestle with the tricky bits of one problem. If a group chooses an inefficient strategy or makes an error, monitor their frustration level, but try to allow them to pursue it in some detail before suggesting there may be an easier method. The first 3 questions (from Schoenfeld) assist students with metacognitive monitoring of their own problem solving process. Whenever possible, allow students to check their own work using graphing technology and/or substitution of their answers rather than checking it for them.

Some good questions to ask groups during this time are:

- "What are you doing?"
- "Why are you doing that?"
- "Is it working?"
- "Are you done?"
- "Have you found values for all the unknowns?"
- "How could you check your work?"

- "Can you graph the problem to check your work?"
- "Can you substitute these numbers back in to check your work?"
- "What would you expect to see on the graph?"

Suggestions for Facilitating Whole Class Discussion

Remind students of their norms for active listening during presentations, respect for presenters and treating mistakes as learning opportunities. Ask the reporters from at least 4-6 groups to share out their work (the reporter role should rotate regularly, even every class period). If not all groups will present, give priority to students or groups who present less often but who have done excellent work, to groups that have tried innovative strategies or made important revisions, and to the most important special cases. When sequencing the presentations, start with easier and/or typical examples. Often, it should work well to simply present the examples you choose in numerical order. Close with an exit ticket like "Explain one way you revised your work or thinking today" or "Use Jorge's method to solve this new problem." You can also create an exit ticket in advance, such as the one attached.