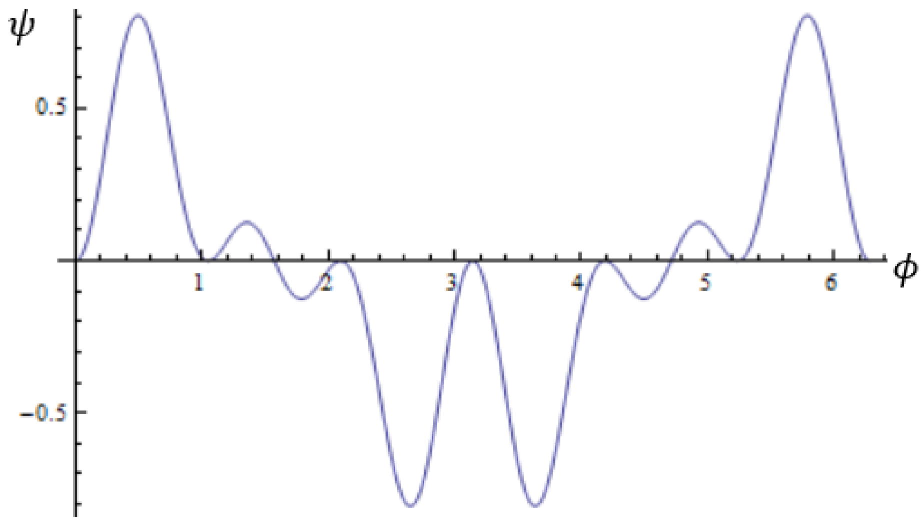
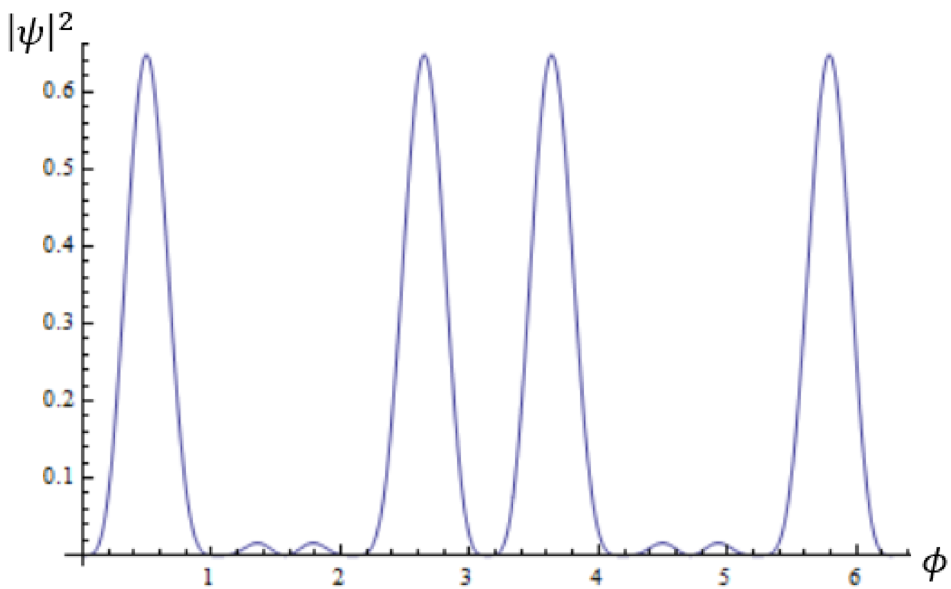
Student handout Consider the following normalized quantum state on a unit ring:

$$\Phi(\phi) = \sqrt{\frac{8}{3\pi r_0}} \sin^2(3\,\phi)\cos(\phi) \tag{1}$$

1. Translate this state into eigenfunction, bra/ket, and matrix representations. Remember that you can use any of these representations in the following calculations.
2. What is the expectation value of L_z in this state?
3. The wave function and it's probability density are plotted below. (I have set $r_0 = 1$ to make the plotting easier). What features of these graphs (if any) tell you the expectation value of L_z in this state?





4. What is the probability that the particle can be found in the region $0 < \phi < \frac{\pi}{4}$? Repeat your calculation in the region $\frac{\pi}{4} < \phi < \frac{3\pi}{4}$?

1 Instructor's Guide

1.1 Student Conversations

- **How to split apart the wavefunction:** There are 2 general ways to split apart a wavefunction like this into eigenstates, and both are part of this course. The first (and easier in this case) is to use Euler's Identity for sine and cosine and write this wavefunction as complex exponentials and then FOIL it all out into things that look like eigenstates. The other option is to do inner products to find the coefficents of the eigenstates like so:

  $$c_m = \langle m|\psi\rangle = \int_0^{2\pi} \frac{1}{\sqrt{2\pi r_0}} e^{-im\phi}\psi(\phi) r_0 d\phi$$

  It is important for students to know both, in particular because the 2nd method will always work and will be exploed heavily in the similar context of Fourier Series in Periodic Systems. This activity is primarily postured towards the 1st method, which will be explore in future problems for the particle on a sphere and hydrogen atom later in this course.

- **Probability v. Probability Density:** Students struggle with the two different ways of finding probability: for discrete and continuous measurements. Most recognize that they need to do an integral for a continuous quantity, but are not sure when to square (before integration or after).

  $$\left|\int \phi_n^*(x)\Psi(x,t)dx\right|^2 \quad vs. \quad \int |\Psi(t)|^2\, dx \tag{2}$$

  In particular, many students will forget to do the squaring for the calculation on the left because $\int \phi_n^*(x)\Psi(x,t)dx$ looks a lot like $\int \Psi^*(x,t)\Psi(x,t)dx$.

- **Noticing symmetry:** Leading up to the wavefunction graphs, students should already know that $\langle L_z\rangle = 0$, so they will be seeking to justify it through these graphs. It is very important that they notice that the axis of the graph are not directly telling them anything about L_z, it is the shape of the wave function that could inform them about that. Physically reasoning that a positive value of L_z would indicate overall rightward movement of the wavefunction and leftward movement for negative values. Then, asking them to think about the probability in one region and if it would change as the wave function was sent to the left or the right at some velocity (the answer is it won't because the ring wraps around continuously). Therefore, they can conclude that since the probabilities wouldn't change with direction of travel, direction doesn't matter for the expectation value of L_z and 0 is the only number which stays the same upon switching signs.

1.2 Wrap-up

- Emphasize both methods of turning the complicated wavefunction into eigenstates (using Euler's Identity and using inner products/integrals to find coefficents).
- Remind students how to find an expectation value, and in partiuclar doing the sum of probability of a measurement times that measurement as a faster method (most will have seen this before, but we want everyone to start seeing that as typically the fastest way).
- How to recognize symmetry in the probability density and that for even eave functions, we can conclude they will have an expectation value of 0.
- If student shave found the probability in a region recently, it isn't necessary to wrap up the last part. Setting up one of the the integrals is likely sufficent.