Student handout For $\ell = 1$, the operators that measure the three components of angular momentum in matrix notation are given by:

$$L_x = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \tag{1}$$

$$L_y = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \tag{2}$$

$$L_z = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{3}$$

Show that:

1. Find the commutator of L_x and L_y.
2. Find the matrix representation of $L^2 = L_x^2 + L_y^2 + L_z^2$.
3. Find the matrix representations of the raising and lowering operators $L_\pm = L_x \pm iL_y$. (Notice that $L_\pm$ are NOT Hermitian and therefore cannot represent observables. They are used as a tool to build one quantum state from another.)
4. Show that $[L_z, L_\pm] = \lambda L_\pm$. Find λ. Interpret this expression as an eigenvalue equation. What is the operator?
5. Let L_+ act on the following three states given in matrix representation.

$$|1,1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad |1,0\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad |1,-1\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{4}$$

   Why is L_+ called a "raising operator"?

1 Instructor's Guide

1.1 Introduction

This activity is meant to lay the foundation of what raising and lowering oporators are and how they can be used. This material will become very important for students' study of symmetry matrices in PH427 and the Quantum Harmonic Oscillator in the Quantum Capstone.

1.2 Student Conversations

At this stage, students will not have seen commutators or done much matrix multiplication in a while, so students may progress lower here than you'd expect. It will be important for the teaching team to be on the look out for groups that are confused at the beginning since some will forget that a commutator can have the form $[A, B] = AB - BA$, which is necessary to progress.

Making sure the teaching team has a good handle on the results of each calculation so they can help trouble shoot errors made during matrix multiplication which are hard to catch in the act and usually can most easilty be inferred from an erronous result (which the students themselves won't usually recognize).

1.3 Wrap-up

It is a good idea to reinforce the patterns seen in orbital angular momentum to their experiences with spin angular momentum, such as that cross product-like relationship between commutators of cartesian directed angular momenta. Then it becomes easy to contrast those patterns with that of the raising and lower operators and emphasize that these are not observables which correspond to measures of angular momentum but a different object entirely.

While their importance should be emphasized for study of periodic systems and the quantum harmonic oscillator, it should also be mentioned these operators will not be a major focus of this course or our study of the Hydrogen atom as we head into the home stretch of the course. This content is largely a very important detour.