Student handout

The operator $\hat{L}_z$ that represents the z-component of angular momentum, the operator $\hat{L}^2$ that represents the total angular momentum, and the operator $\hat{H}$ that represents the energy for the rigid rotor (a particle confined to the unit sphere) have eigenvalues given by

$$\hat{L}_z |\ell, m\rangle = m\hbar |\ell, m\rangle \tag{1}$$

$$\hat{L}^2 |\ell, m\rangle = \ell(\ell+1)\hbar^2 |\ell, m\rangle \tag{2}$$

$$\hat{H} |\ell, m\rangle = \frac{\hbar^2}{2I} \ell(\ell+1) |\ell, m\rangle \tag{3}$$

Find the matrix representations for these operators.

1 Instructor's Guide

1.1 Introduction

This is an activity that is meant to make students think critically about indexing and the limits of the quantum numbers on the sphere.

1.2 Student Conversations

At this stage, several students will still be a bit confused on the form of the operators. You may see folks writing out column vectors and if that's the case, having a conversation about beasts is necessary, emphasizing the vectors in quantum mechanics are states and operators are square matrices. The teaching team should be instructed to quickly look for this type of error and correct people early on so all the groups can get to the point of wrestling with the structure of the matrix.

For the ring, there wasn't an obvious starting point to start indexing since the only quantum number was m and it went between positive and negative infinity. Here, the "obvious" place to start is at $\ell = 0$ and $m = 0$. This is often difficult for students to grasp at this stage because one of the central reasons we do this is because these number correspond to the ground state energy and students will not have the background to understand the importance of the ground state in full, that inituition will usually solidify in studying hydrogen or the quantum harmonic oscillator. This is a great time to emphasize this to students who are stuggling to move forward once they've started thinking about their matrix.

Another stumbling point will be if they should index ℓ for each m or m for each ℓ. This is cleared up more quickly since the boundaries for m involve ℓ, so they need to pick ℓ then let the m's follow. There is still a matter of choice to index from negative to positive or positive to negative m values, and while it is a choice, McIntyre and our experiences on the ring say we should start with negative m values and go to positive ones.

1.3 Wrap-up

The wrap up should emphaize the thought process one goes through when choosing how to index numbers like this. Students need to see that modeled by a professional several times and the transpacancy of

when you're making a choice based on logic vs. usefullness vs. simply conventional should be stated explicitly. It is useful and logical to begin indexing with the ground state but the choice of whether to start with positive or negative values of m is conventional.

It may be important if the class is scattered near the beginning to give them a short mid-problem wrap up discussing beasts, why operators are square matrices, and how you can start thinking about indexing. Students should take a while puzzling through this, but if all groups aren't getting any traction, opt for a class wide discussion to get them started.