Student handout

For the state

$$|\Psi\rangle = \sqrt{\frac{7}{10}}|2,1,0\rangle + \sqrt{\frac{1}{10}}|3,2,1\rangle + i\sqrt{\frac{2}{10}}|3,1,1\rangle$$

1. Convert the state to matrix notation.
2. Find $\mathcal{P}(L_z = 0\hbar)$
3. Find $\langle E\rangle$

1 Instructor's Guide

1.1 Students' Task

Students are asked to find eigenvalues, probabilities, and expectation values for H, L^2, and L_z for a superposition of $|n\ell m\rangle$ states. This can be done on small whiteboards or with the students working in groups on large whiteboards.

Students then work together in small groups to find the matrices that correspond to H, L^2, and L_z and to redo $\langle E\rangle$ in matrix notation

1.2 Student Conversations

- In writing matrices, this is the often the first time that they are only writing a matrix for a subset of the space, so questions about basis, degeneracy, and order of the indices crop up.
- Make sure students are showing the calculations in matrix notation and doing them that way as practice and not doing it by reading probabilities off or using kets.
- For the expectation value, encourage them to use matrix notation so they can see how it works again, but if they want to use the probabilities times eigenvalues way stimulate a conversation about when one is easier vs. another and include that as part of the wrap up.
- Even at this stage, the most common error during calculations is likely to be forgetting to complex conjugate bras. Watch for this.

1.3 Wrap-up

- If the students use small whiteboards to write $\hat{H}$, $\hat{L}^2$, and $\hat{L}_z$ in matrix notation for this state, you can often find that people have used different ordering systems and by using several different examples, you can highlight several issues:
  - the arbitrariness of the order when there is degeneracy,
  - importance of being consistent with order between operators and vectors,

  - what the "typical" order is for hydrogen states, and
  - matrix notation is not as unwieldy as it seems if you have a small enough subspace.

- You can also highlight the implicit basis ($|n\ell m\rangle$) and reiterate the fact that all three matrices are diagonal in this basis (i.e. they share eigenstates).

This is also a good time to talk about the different ways of finding expectation values and when each is appropriate.

1.4 Extension

This same state us used to do calculations in the ket representation in Quantum Calculations on the Hydrogen Atom, which ideally should procede this, but could be done in either order.